

thm\_2Equotient\_list\_2ENIL\_\_RSP  
(TMVnKjCb25JDyBuurbrNny9oWf78d7jXs3L)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

**Definition 3** We define `c_2Ebool_2ET` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A-27a}))))$

**Definition 5** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V2t \in 2. V2t))))$

**Definition 6** We define `c_2Equotient_2EQUOTIENT` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0R \in ((2^{A-27a})^{A-27a}). \lambda V0t \in 2. V0t$

**Definition 7** We define `c_2Ebool_2EF` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V0t \in 2. V0t))$ .

Let `ty_2Elist_2Elist` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty\_2Elist\_2Elist } A0) \quad (1)$$

Let `c_2Elist_2ECONS` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c\_2Elist\_2ECONS } A. 27a \in (((\text{ty\_2Elist\_2Elist } A. 27a)^{(\text{ty\_2Elist\_2Elist } A. 27a)})^{A. 27a}) \quad (2)$$

Let `c_2Elist_2ENIL` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c\_2Elist\_2ENIL } A. 27a \in (\text{ty\_2Elist\_2Elist } A. 27a) \quad (3)$$

Let `c_2Elist_2ELIST__REL` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c\_2Elist\_2ELIST__REL } A. 27a \ A. 27b \in (((2^{(\text{ty\_2Elist\_2Elist } A. 27b)})^{(\text{ty\_2Elist\_2Elist } A. 27a)})^{(2^{A-27b})^{A-27a}}) \quad (4)$$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27b})^{A\_27a}). (\forall V1a \in A\_27a. (\forall V2as \in \\
& \quad \quad (ty\_2Elist\_2Elist\ A\_27a). (\forall V3b \in A\_27b. (\forall V4bs \in \\
& \quad \quad \quad (ty\_2Elist\_2Elist\ A\_27b). (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL \\
& \quad \quad \quad \quad A\_27a\ A\_27b)\ V0R)\ (c\_2Elist\_2ENIL\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27b)))) \Leftrightarrow \\
& \quad \quad \quad \quad True) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b)\ V0R) \\
& \quad \quad \quad \quad (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a)\ V2as))\ (c\_2Elist\_2ENIL\ A\_27b)))) \Leftrightarrow \\
& \quad \quad \quad \quad False) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b)\ V0R) \\
& \quad \quad \quad \quad (c\_2Elist\_2ENIL\ A\_27a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V3b)\ V4bs)))) \Leftrightarrow \\
& \quad \quad \quad \quad False) \wedge ((p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL\ A\_27a\ A\_27b)\ V0R) \\
& \quad \quad \quad \quad (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a)\ V2as))\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& \quad \quad \quad \quad A\_27b)\ V3b)\ V4bs)))) \Leftrightarrow ((p\ (ap\ (ap\ V0R\ V1a)\ V3b)) \wedge (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL \\
& \quad \quad \quad \quad A\_27a\ A\_27b)\ V0R)\ V2as)\ V4bs))))))))) \\
& \tag{6}
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1abs \in (A\_27b^{A\_27a}). \\
& \quad \quad (\forall V2rep \in (A\_27a^{A\_27b}). ((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad \quad \quad \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Elist\_2ELIST\_REL \\
& \quad \quad \quad \quad A\_27a\ A\_27a)\ V0R)\ (c\_2Elist\_2ENIL\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27a)))))) \\
& \tag{6}
\end{aligned}$$