

thm_2Equotient_list_2EREVERSE__RSP
(TMQmgBfHVeuJqiicd4yVD2YhRgVkgGktbzW)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (2)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (3)$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EREVERSE A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)}) \quad (4)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (5)$$

Let $c_2Elist_2ELIST_REL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2ELIST_REL\ A_27a\ A_27b \in (((2^{(ty_2Elist_2Elist\ A_27b)})^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27b})^{A_27a}}) \quad (6)$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Definition 8 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}).(((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist\ A_27a).(p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a\ V2h\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist\ A_27a).(p\ (ap\ V0P\ V3l)))))) \quad (13)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27b})^{A_27a}). (\forall V1a \in A_27a. (\forall V2as \in \\
& \quad (ty_2Elist_2Elist\ A_27a). (\forall V3b \in A_27b. (\forall V4bs \in \\
& \quad (ty_2Elist_2Elist\ A_27b). ((p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL \\
& \quad A_27a\ A_27b)\ V0R)\ (c_2Elist_2ENIL\ A_27a))\ (c_2Elist_2ENIL\ A_27b)))) \Leftrightarrow \\
& \quad True) \wedge (((p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL\ A_27a\ A_27b)\ V0R) \\
& \quad (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1a)\ V2as))\ (c_2Elist_2ENIL\ A_27b)))) \Leftrightarrow \\
& \quad False) \wedge (((p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL\ A_27a\ A_27b)\ V0R) \\
& \quad (c_2Elist_2ENIL\ A_27a))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ V3b)\ V4bs)))) \Leftrightarrow \\
& \quad False) \wedge (((p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL\ A_27a\ A_27b)\ V0R) \\
& \quad (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1a)\ V2as))\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27b)\ V3b)\ V4bs)))) \Leftrightarrow ((p\ (ap\ (ap\ V0R\ V1a)\ V3b)) \wedge (p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL \\
& \quad A_27a\ A_27b)\ V0R)\ V2as)\ V4bs))))))))) \\
& \hspace{15em} (14)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2EREVERSE\ A_27a) \\
& \quad (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27a)) \wedge (\forall V0h \in \\
& \quad A_27a. (\forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2EREVERSE \\
& \quad A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad A_27a)\ (ap\ (c_2Elist_2EREVERSE\ A_27a)\ V1t))\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27a)\ V0h)\ (c_2Elist_2ENIL\ A_27a)))))) \\
& \hspace{15em} (15)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b)^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a)^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3l1 \in (ty_2Elist_2Elist \\
& \quad A_27a). (\forall V4l2 \in (ty_2Elist_2Elist\ A_27a). (\forall V5m1 \in \\
& \quad (ty_2Elist_2Elist\ A_27a). (\forall V6m2 \in (ty_2Elist_2Elist\ A_27a). \\
& \quad (((p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL\ A_27a\ A_27a)\ V0R)\ V3l1)\ V4l2)) \wedge \\
& \quad (p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL\ A_27a\ A_27a)\ V0R)\ V5m1)\ V6m2)))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (ap\ (c_2Elist_2ELIST_REL\ A_27a\ A_27a)\ V0R)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad A_27a)\ V3l1)\ V5m1))\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V4l2)\ V6m2))))))))) \\
& \hspace{15em} (16)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b)^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a)^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3l1 \in (ty_2Elist_2Elist \\
& \quad A_27a). (\forall V4l2 \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ (ap \\
& \quad (c_2Elist_2ELIST_REL\ A_27a\ A_27a)\ V0R)\ V3l1)\ V4l2)) \Rightarrow (p\ (ap\ (ap \\
& \quad (ap\ (c_2Elist_2ELIST_REL\ A_27a\ A_27a)\ V0R)\ (ap\ (c_2Elist_2EREVERSE \\
& \quad A_27a)\ V3l1))\ (ap\ (c_2Elist_2EREVERSE\ A_27a)\ V4l2))))))))) \\
& \hspace{15em}
\end{aligned}$$