

thm\_2Equotient\_\_option\_2EIS\_\_NONE\_\_PRS  
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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (P \Rightarrow Q)$  of type  $\iota$ .

**Definition 3** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A_27a}))))$

**Definition 5** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V2t \in 2. V2t))))$

**Definition 6** We define `c_2Equotient_2EQUOTIENT` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1R \in ((2^{A_27b})^{A_27b}). \text{inj\_o } (V0R = V1R)$

**Definition 7** We define `c_2Ebool_2E_2F` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V0t \in 2. V0t))$ .

**Definition 8** We define `c_2Ebool_2E_27E` to be  $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D\_3D\_3E } V0t) (\text{c\_2Ebool\_2E\_2F } 2))))$

**Definition 9** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define `c_2Ebool_2E_3F` to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P (\text{ap } (\text{c\_2Emin\_2E\_40 } A_27a))))$

**Definition 11** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V2t \in 2. V2t))))$

Let `ty_2Eoption_2Eoption` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty\_2Eoption\_2Eoption } A0) \quad (1)$$

Let `c_2Eoption_2EOPTION_MAP` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \text{c\_2Eoption\_2EOPTION\_MAP } A_27a \ A_27b \in (((\text{ty\_2Eoption\_2Eoption } A_27b)^{(\text{ty\_2Eoption\_2Eoption } A_27a)})^{(A_27b^{A_27a})}) \quad (2)$$

Let `ty_2Eone_2Eone` :  $\iota$  be given. Assume the following.

$$\text{nonempty } (\text{ty\_2Eone\_2Eone}) \quad (3)$$

**Definition 12** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone\_2$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Esum\_2Esum A0 A1) \quad (4)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Esum\_2EABS\_sum A\_27a A\_27b \in ((ty\_2Esum\_2Esum A\_27a A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (5)$$

**Definition 13** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap (c\_2Esum\_2EABS$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (6)$$

**Definition 14** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eoption\_2Eoption\_ABS A\_27a) (c$

**Definition 15** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27a. (ap (c\_2Esum\_2EABS$

**Definition 16** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. (ap (c\_2Eoption\_2Eoption\_ABS$

Let  $c\_2Eoption\_2EIS\_NONE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Eoption\_2EIS\_NONE A\_27a \in (2^{(ty\_2Eoption\_2Eoption A\_27a)}) \quad (7)$$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow ((p V0t) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))))) \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption A\_27a). ((V0opt = (c\_2Eoption\_2ENONE A\_27a)) \vee (\exists V1x \in A\_27a. (V0opt = (ap (c\_2Eoption\_2ESOME A\_27a) V1x)))))) \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP \\ A\_27a\ A\_27b)\ V0f)\ (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V1x)) = (ap\ (c\_2Eoption\_2ESOME \\ A\_27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP \\ A\_27a\ A\_27b)\ V2f)\ (c\_2Eoption\_2ENONE\ A\_27a)) = (c\_2Eoption\_2ENONE \\ A\_27b)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a.((p\ (ap\ (c\_2Eoption\_2EIS\_NONE \\ A\_27a)\ (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V0x))) \Leftrightarrow False)) \wedge ((p\ (ap\ ( \\ c\_2Eoption\_2EIS\_NONE\ A\_27a)\ (c\_2Eoption\_2ENONE\ A\_27a))) \Leftrightarrow True)) \end{aligned} \quad (13)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\ (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x \in (ty\_2Eoption\_2Eoption \\ A\_27b).((p\ (ap\ (c\_2Eoption\_2EIS\_NONE\ A\_27b)\ V3x)) \Leftrightarrow (p\ (ap\ (c\_2Eoption\_2EIS\_NONE \\ A\_27a)\ (ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP\ A\_27b\ A\_27a)\ V2rep)\ V3x)))))))))) \end{aligned}$$