

thm_2Equotient__option_2EIS__NONE__RSP
(TML89Cq2osdC9UyPNakuRMfY5cp3FqNg4dT)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Definition 3 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Definition 5 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 6 We define `c_2Equotient_2EQUOTIENT` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a})^{A_27b}$

Definition 7 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 8 We define `c_2Ebool_2E_2E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2E_2F})))$

Definition 9 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A_27a))))$

Definition 11 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Let `ty_2Eoption_2Eoption` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Eoption_2Eoption } A0) \quad (1)$$

Let `c_2Eoption_2EIS__NONE` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Eoption_2EIS_NONE } A_27a \in (2^{(\text{ty_2Eoption_2Eoption } A_27a)}) \quad (2)$$

Let `ty_2Eone_2Eone` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Eone_2Eone} \quad (3)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (4)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (5)$$

Definition 12 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (6)$$

Definition 13 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 14 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone. V0x))$

Definition 15 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 16 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (\lambda V0x \in A_27a. \perp))$

Definition 17 We define $c_2Eoption_2EOPTREL$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). \lambda V1x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V1x)$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & (p\ V0t)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption\ A.27a). ((V0opt = (c_2Eoption_2ENONE\ A.27a)) \vee (\exists V1x \in A.27a. (V0opt = (ap\ (c_2Eoption_2ESOME\ A.27a)\ V1x)))))) \quad (12)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0x \in A.27a. ((p\ (ap\ (c_2Eoption_2EIS_NONE\ A.27a)\ (ap\ (c_2Eoption_2ESOME\ A.27a)\ V0x))) \Leftrightarrow False)) \wedge ((p\ (ap\ (c_2Eoption_2EIS_NONE\ A.27a)\ (c_2Eoption_2ENONE\ A.27a))) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0R \in ((2^{A.27a})^{A.27a}). \\ & (\forall V1x \in A.27a. (\forall V2y \in A.27a. (((p\ (ap\ (ap\ (ap\ (c_2Eoption_2EOPTREL\ A.27a\ A.27a)\ V0R)\ (c_2Eoption_2ENONE\ A.27a))\ (c_2Eoption_2ENONE\ A.27a))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ (ap\ (c_2Eoption_2EOPTREL\ A.27a\ A.27a)\ V0R)\ (ap\ (c_2Eoption_2ESOME\ A.27a)\ V1x))\ (c_2Eoption_2ENONE\ A.27a))) \Leftrightarrow \\ & False) \wedge (((p\ (ap\ (ap\ (ap\ (c_2Eoption_2EOPTREL\ A.27a\ A.27a)\ V0R)\ (c_2Eoption_2ENONE\ A.27a))\ (ap\ (c_2Eoption_2ESOME\ A.27a)\ V2y))) \Leftrightarrow \\ & False) \wedge ((p\ (ap\ (ap\ (ap\ (c_2Eoption_2EOPTREL\ A.27a\ A.27a)\ V0R)\ (ap\ (c_2Eoption_2ESOME\ A.27a)\ V1x))\ (ap\ (c_2Eoption_2ESOME\ A.27a)\ V2y))) \Leftrightarrow (p\ (ap\ (ap\ V0R\ V1x)\ V2y)))))))))) \quad (14) \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0R \in ((2^{A.27a})^{A.27a}). (\forall V1abs \in (A.27b^{A.27a}). \\ & (\forall V2rep \in (A.27a^{A.27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A.27a\ A.27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x \in (ty_2Eoption_2Eoption\ A.27a). (\forall V4y \in (ty_2Eoption_2Eoption\ A.27a). ((p\ (ap\ (ap\ (ap\ (c_2Eoption_2EOPTREL\ A.27a\ A.27a)\ V0R)\ V3x)\ V4y)) \Rightarrow ((p\ (ap\ (c_2Eoption_2EIS_NONE\ A.27a)\ V3x)) \Leftrightarrow (p\ (ap\ (c_2Eoption_2EIS_NONE\ A.27a)\ V4y)))))))))) \end{aligned}$$