

thm_2Equotient__option_2EIS__SOME__PRS
(TMPS7WkAtCG3dz9N6JcXydBBuUVr9YGd6fE)

October 26, 2020

Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 3 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Definition 5 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Definition 6 We define `c_2Equotient_2EQUOTIENT` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1R \in ((2^{A_27b})^{A_27b}). \text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0R) V1R$

Definition 7 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2. V0t)$.

Definition 8 We define `c_2Ebool_2E_27E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\text{c_2Ebool_2E_2F } V0t))))$

Definition 9 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } V0P))))$

Definition 11 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Let `ty_2Eoption_2Eoption` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Eoption_2Eoption } A0) \quad (1)$$

Let `c_2Eoption_2EOPTION_MAP` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \text{c_2Eoption_2EOPTION_MAP } A_27a \ A_27b \in (((\text{ty_2Eoption_2Eoption } A_27b)^{(\text{ty_2Eoption_2Eoption } A_27a)})^{(A_27b^{A_27a})}) \quad (2)$$

Let `ty_2Eone_2Eone` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Eone_2Eone} \quad (3)$$

Definition 12 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone_2$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (4)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (5)$$

Definition 13 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (6)$$

Definition 14 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c$

Definition 15 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_2Esum_2EABS$

Definition 16 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_ABS$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2EIS_SOME A_27a \in (2^{(ty_2Eoption_2Eoption A_27a)}) \quad (7)$$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow ((p V0t) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))))) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption A_27a). ((V0opt = (c_2Eoption_2ENONE A_27a)) \vee (\exists V1x \in A_27a. (V0opt = (ap (c_2Eoption_2ESOME A_27a) V1x)))))) \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0f \in (A_27b^{A_27a}).(\forall V1x \in A_27a.((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ & A_27a\ A_27b)\ V0f)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)) = (ap\ (c_2Eoption_2ESOME \\ & A_27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ & A_27a\ A_27b)\ V2f)\ (c_2Eoption_2ENONE\ A_27a)) = (c_2Eoption_2ENONE \\ & A_27b)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a.((p\ (ap\ (c_2Eoption_2EIS_SOME \\ & A_27a)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x))) \Leftrightarrow True)) \wedge ((p\ (ap\ (c_2Eoption_2EIS_SOME \\ & A_27a)\ (c_2Eoption_2ENONE\ A_27a))) \Leftrightarrow False)) \end{aligned} \tag{13}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\ & (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x \in (ty_2Eoption_2Eoption \\ & A_27b).((p\ (ap\ (c_2Eoption_2EIS_SOME\ A_27b)\ V3x)) \Leftrightarrow (p\ (ap\ (c_2Eoption_2EIS_SOME \\ & A_27a)\ (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ A_27b\ A_27a)\ V2rep)\ V3x)))))))))) \end{aligned}$$