

thm\_2Equotient\_\_option\_2EOPTION\_\_EQUIV  
(TM-  
Nfdy1yP6Dsk3yaWYW3V8wfMuFtMXCiVKS)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Emin\_2E\_240$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_23F$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_240 A$

**Definition 9** We define  $c\_2Ebool\_2E\_25C\_2E\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Eoption\_2Eoption A0) \quad (1)$$

**Definition 10** We define  $c\_2Equotient\_2EEQUIV$  to be  $\lambda A.\lambda a : \iota.\lambda V0E \in ((2^{A-27a})^{A-27a}).(ap (c\_2Ebool\_2E\_25C\_2E\_2F$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty ty\_2Eone\_2Eone \quad (2)$$

**Definition 11** We define  $c\_2Ebool\_2E\_25C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (3)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (4)$$

**Definition 12** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b)\ V0e)$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (5)$$

**Definition 13** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ V0x)$

**Definition 14** We define  $c\_2Eone\_2Eone$  to be  $(ap\ (c\_2Emin\_2E40\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2Eone.\ V0x))$

**Definition 15** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b)\ V0e)$

**Definition 16** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ (c\_2Eone\_2Eone))$

**Definition 17** We define  $c\_2Eoption\_2EOPTREL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27b})^{A\_27a}).\lambda V1x \in A\_27b.(ap\ (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ V1x)$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (8)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& p V0t))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption \\
& A\_27a).(V0opt = (c\_2Eoption\_2ENONE A\_27a)) \vee (\exists V1x \in A\_27a. \\
& (V0opt = (ap (c\_2Eoption\_2ESOME A\_27a) V1x))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& (\forall V1x \in A\_27a. (\forall V2y \in A\_27a. (((p (ap (ap (ap (c\_2Eoption\_2EOPTREL \\
& A\_27a A\_27a) V0R) (c\_2Eoption\_2ENONE A\_27a)) (c\_2Eoption\_2ENONE \\
& A\_27a))) \Leftrightarrow True) \wedge (((p (ap (ap (ap (c\_2Eoption\_2EOPTREL A\_27a A\_27a) \\
& V0R) (ap (c\_2Eoption\_2ESOME A\_27a) V1x)) (c\_2Eoption\_2ENONE A\_27a))) \Leftrightarrow \\
& False) \wedge (((p (ap (ap (ap (c\_2Eoption\_2EOPTREL A\_27a A\_27a) V0R) \\
& (c\_2Eoption\_2ENONE A\_27a)) (ap (c\_2Eoption\_2ESOME A\_27a) V2y))) \Leftrightarrow \\
& False) \wedge ((p (ap (ap (ap (c\_2Eoption\_2EOPTREL A\_27a A\_27a) V0R) ( \\
& ap (c\_2Eoption\_2ESOME A\_27a) V1x)) (ap (c\_2Eoption\_2ESOME A\_27a) \\
& V2y))) \Leftrightarrow (p (ap (ap V0R V1x) V2y)))))))))
\end{aligned} \tag{13}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& ((p (ap (c\_2Equotient\_2EEQUIV A\_27a) V0R)) \Rightarrow (p (ap (c\_2Equotient\_2EEQUIV \\
& (ty\_2Eoption\_2Eoption A\_27a)) (ap (c\_2Eoption\_2EOPTREL A\_27a \\
& A\_27a) V0R))))
\end{aligned}$$