

# thm\_2Equotient\_option\_2EOPTION\_QUOTIENT (TMHMB8TxfH25wF4CgiNcTZm5Q58B1AZpJLA)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x)) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2E_3F` to be  $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } V0P \text{ (ap } (c_2Emin_2E_40 \ A_{27a} \ P)) \text{ of type } \iota$ .

**Definition 4** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota$ .

**Definition 5** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2. V0x)) \ (\lambda V1x \in 2. V1x))$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_21` to be  $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A_{27a}})) \ P)) \text{ of type } \iota$ .

**Definition 7** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V2t \in 2. V2t)) \text{ of type } \iota$ .

Let `ty_2Eoption_2Eoption` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty\_2Eoption\_2Eoption } A0) \quad (1)$$

Let `c_2Eoption_2EOPTION_JOIN` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow c_2Eoption_2EOPTION\_JOIN \ A_{27a} \in ((\text{ty\_2Eoption\_2Eoption } A_{27a})^{(\text{ty\_2Eoption\_2Eoption } (\text{ty\_2Eoption\_2Eoption } A_{27a}))}) \quad (2)$$

Let `c_2Eoption_2EOPTION_MAP` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow c_2Eoption_2EOPTION\_MAP \ A_{27a} \ A_{27b} \in (((\text{ty\_2Eoption\_2Eoption } A_{27b})^{(\text{ty\_2Eoption\_2Eoption } A_{27a})})^{(A_{27b}^{A_{27a}})}) \quad (3)$$

Let  $c\_2Eoption\_2Eoption\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a}})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (4)$$

Let  $c\_2Eoption\_2EIS\_NONE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_NONE\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (5)$$

Let  $c\_2Eoption\_2EIS\_SOME : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (6)$$

**Definition 8** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 9** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$ .

Let  $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (7)$$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$ .

**Definition 11** We define  $c\_2Equotient\_2EQUOTIENT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27b}).\lambda V0R$ .

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (8)$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (9)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (10)$$

**Definition 12** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b)\ V0e))$ .

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (11)$$

**Definition 13** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap (c\_2Eoption\_2Eoption\_2ESOME A\_27a) (V0x))$

**Definition 14** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone. V0x))$

**Definition 15** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap (c\_2Esum\_2EABS A\_27a A\_27b) (V0e))$

**Definition 16** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap (c\_2Eoption\_2Eoption\_2ENONE A\_27a) (V0x))$

**Definition 17** We define  $c\_2Eoption\_2EOPTREL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27b})^{A\_27a}).\lambda V1x \in A\_27b.(ap (c\_2Eoption\_2EOPTREL A\_27a A\_27b) (V0R V1x))$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (15)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (16)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption A\_27a).((V0opt = (c\_2Eoption\_2ENONE A\_27a)) \vee (\exists V1x \in A\_27a.(V0opt = (ap (c\_2Eoption\_2ESOME A\_27a) V1x)))))) \quad (19)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP \\
& A\_27a\ A\_27b)\ V0f)\ (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V1x)) = (ap\ (c\_2Eoption\_2ESOME \\
& A\_27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP \\
& A\_27a\ A\_27b)\ V2f)\ (c\_2Eoption\_2ENONE\ A\_27a)) = (c\_2Eoption\_2ENONE \\
& A\_27b))))
\end{aligned}$$

(20)

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow ( \\
& \quad \forall V0e \in A_{.27b}.(\forall V1f \in (A_{.27b}^{A_{.27a}}).(\forall V2e \in ( \\
& \quad \quad ty\_2Eoption\_2Eoption\ A_{.27a}).(\forall V3x \in A_{.27a}.(\forall V4y \in \\
& \quad \quad \quad A_{.27a}.(((ap\ (c\_2Eoption\_2ESOME\ A_{.27a})\ V3x) = (ap\ (c\_2Eoption\_2ESOME \\
& \quad \quad \quad A_{.27a})\ V4y)) \Leftrightarrow (V3x = V4y)))) \wedge ((\forall V5x \in A_{.27a}.((ap\ (c\_2Eoption\_2ETHE \\
& \quad \quad \quad A_{.27a})\ (ap\ (c\_2Eoption\_2ESOME\ A_{.27a})\ V5x)) = V5x)) \wedge ((\forall V6x \in \\
& \quad \quad \quad A_{.27a}.(\neg((c\_2Eoption\_2ENONE\ A_{.27a}) = (ap\ (c\_2Eoption\_2ESOME \\
& \quad \quad \quad A_{.27a})\ V6x)))) \wedge ((\forall V7x \in A_{.27a}.(\neg((ap\ (c\_2Eoption\_2ESOME \\
& \quad \quad \quad A_{.27a})\ V7x) = (c\_2Eoption\_2ENONE\ A_{.27a})))) \wedge ((\forall V8x \in A_{.27a}. \\
& \quad \quad \quad ((p\ (ap\ (c\_2Eoption\_2EIS\_SOME\ A_{.27a})\ (ap\ (c\_2Eoption\_2ESOME \\
& \quad \quad \quad A_{.27a})\ V8x))) \Leftrightarrow True)) \wedge ((p\ (ap\ (c\_2Eoption\_2EIS\_SOME\ A_{.27a}) \\
& \quad \quad \quad (c\_2Eoption\_2ENONE\ A_{.27a}))) \Leftrightarrow False) \wedge ((\forall V9x \in (ty\_2Eoption\_2Eoption \\
& \quad \quad \quad A_{.27a}).((p\ (ap\ (c\_2Eoption\_2EIS\_NONE\ A_{.27a})\ V9x)) \Leftrightarrow (V9x = (c\_2Eoption\_2ENONE \\
& \quad \quad \quad A_{.27a})))) \wedge ((\forall V10x \in (ty\_2Eoption\_2Eoption\ A_{.27a}).(\neg \\
& \quad \quad \quad (p\ (ap\ (c\_2Eoption\_2EIS\_SOME\ A_{.27a})\ V10x))) \Leftrightarrow (V10x = (c\_2Eoption\_2ENONE \\
& \quad \quad \quad A_{.27a})))) \wedge ((\forall V11x \in (ty\_2Eoption\_2Eoption\ A_{.27a}).((p \\
& \quad \quad \quad (ap\ (c\_2Eoption\_2EIS\_SOME\ A_{.27a})\ V11x)) \Rightarrow ((ap\ (c\_2Eoption\_2ESOME \\
& \quad \quad \quad A_{.27a})\ (ap\ (c\_2Eoption\_2ETHE\ A_{.27a})\ V11x)) = V11x))) \wedge ((\forall V12x \in \\
& \quad \quad \quad (ty\_2Eoption\_2Eoption\ A_{.27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE \\
& \quad \quad \quad A_{.27a}\ (ty\_2Eoption\_2Eoption\ A_{.27a})\ V12x)\ (c\_2Eoption\_2ENONE \\
& \quad \quad \quad A_{.27a})\ (c\_2Eoption\_2ESOME\ A_{.27a}) = V12x)) \wedge ((\forall V13x \in ( \\
& \quad \quad \quad ty\_2Eoption\_2Eoption\ A_{.27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE \\
& \quad \quad \quad A_{.27a}\ (ty\_2Eoption\_2Eoption\ A_{.27a})\ V13x)\ V13x)\ (c\_2Eoption\_2ESOME \\
& \quad \quad \quad A_{.27a}) = V13x)) \wedge ((\forall V14x \in (ty\_2Eoption\_2Eoption\ A_{.27a}). \\
& \quad \quad \quad ((p\ (ap\ (c\_2Eoption\_2EIS\_NONE\ A_{.27a})\ V14x)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE \\
& \quad \quad \quad A_{.27a}\ A_{.27b})\ V14x)\ V0e)\ V1f) = V0e))) \wedge ((\forall V15x \in (ty\_2Eoption\_2Eoption \\
& \quad \quad \quad A_{.27a}).((p\ (ap\ (c\_2Eoption\_2EIS\_SOME\ A_{.27a})\ V15x)) \Rightarrow ((ap\ (ap \\
& \quad \quad \quad (ap\ (c\_2Eoption\_2Eoption\_CASE\ A_{.27a}\ A_{.27b})\ V15x)\ V0e)\ V1f) = ( \\
& \quad \quad \quad \quad ap\ V1f\ (ap\ (c\_2Eoption\_2ETHE\ A_{.27a})\ V15x)))) \wedge ((\forall V16x \in \\
& \quad \quad \quad (ty\_2Eoption\_2Eoption\ A_{.27a}).((p\ (ap\ (c\_2Eoption\_2EIS\_SOME \\
& \quad \quad \quad A_{.27a})\ V16x)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE\ A_{.27a}\ ( \\
& \quad \quad \quad ty\_2Eoption\_2Eoption\ A_{.27a})\ V16x)\ V2e)\ (c\_2Eoption\_2ESOME\ A_{.27a})) = \\
& \quad \quad \quad \quad V16x))) \wedge ((\forall V17v \in A_{.27b}.(\forall V18f \in (A_{.27b}^{A_{.27a}}). \\
& \quad \quad \quad (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE\ A_{.27a}\ A_{.27b})\ (c\_2Eoption\_2ENONE \\
& \quad \quad \quad A_{.27a})\ V17v)\ V18f) = V17v))) \wedge ((\forall V19x \in A_{.27a}.(\forall V20v \in \\
& \quad \quad \quad A_{.27b}.(\forall V21f \in (A_{.27b}^{A_{.27a}}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE \\
& \quad \quad \quad A_{.27a}\ A_{.27b})\ (ap\ (c\_2Eoption\_2ESOME\ A_{.27a})\ V19x))\ V20v)\ V21f) = \\
& \quad \quad \quad \quad (ap\ V21f\ V19x)))) \wedge ((\forall V22f \in (A_{.27b}^{A_{.27a}}).(\forall V23x \in \\
& \quad \quad \quad A_{.27a}.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP\ A_{.27a}\ A_{.27b})\ V22f)\ ( \\
& \quad \quad \quad \quad ap\ (c\_2Eoption\_2ESOME\ A_{.27a})\ V23x)) = (ap\ (c\_2Eoption\_2ESOME\ A_{.27b}) \\
& \quad \quad \quad \quad (ap\ V22f\ V23x)))))) \wedge ((\forall V24f \in (A_{.27b}^{A_{.27a}}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP \\
& \quad \quad \quad A_{.27a}\ A_{.27b})\ V24f)\ (c\_2Eoption\_2ENONE\ A_{.27a})) = (c\_2Eoption\_2ENONE \\
& \quad \quad \quad A_{.27b}))) \wedge (((ap\ (c\_2Eoption\_2EOPTION\_JOIN\ A_{.27a})\ (c\_2Eoption\_2ENONE \\
& \quad \quad \quad (ty\_2Eoption\_2Eoption\ A_{.27a}))) = (c\_2Eoption\_2ENONE\ A_{.27a})) \wedge \\
& \quad \quad \quad (\forall V25x \in (ty\_2Eoption\_2Eoption\ A_{.27a}).((ap\ (c\_2Eoption\_2EOPTION\_JOIN \\
& \quad \quad \quad A_{.27a})\ (ap\ (c\_2Eoption\_2ESOME\ (ty\_2Eoption\_2Eoption\ A_{.27a}) \\
& \quad \quad \quad \quad V25x))) \neq V25x))))))))))))))))))))))
\end{aligned}$$

(21)

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3a \in A\_27b.((ap\ V1abs \\
& \quad (ap\ V2rep\ V3a)) = V3a))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3a \in A\_27b.(p\ (ap\ (ap \\
& \quad V0R\ (ap\ V2rep\ V3a))\ (ap\ V2rep\ V3a))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3r \in A\_27a.(\forall V4s \in \\
& A\_27a.((p\ (ap\ (ap\ V0R\ V3r)\ V4s)) \Leftrightarrow ((p\ (ap\ (ap\ V0R\ V3r)\ V3r)) \wedge ((p\ (ap \\
& \quad (ap\ V0R\ V4s)\ V4s)) \wedge ((ap\ V1abs\ V3r) = (ap\ V1abs\ V4s))))))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(((p\ (ap\ (ap\ (ap\ (c\_2Eoption\_2EOPTREL \\
& \quad A\_27a\ A\_27a)\ V0R)\ (c\_2Eoption\_2ENONE\ A\_27a))\ (c\_2Eoption\_2ENONE \\
& \quad A\_27a))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Eoption\_2EOPTREL\ A\_27a\ A\_27a) \\
& \quad V0R)\ (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V1x))\ (c\_2Eoption\_2ENONE\ A\_27a))) \Leftrightarrow \\
& \quad False) \wedge (((p\ (ap\ (ap\ (ap\ (c\_2Eoption\_2EOPTREL\ A\_27a\ A\_27a)\ V0R) \\
& \quad (c\_2Eoption\_2ENONE\ A\_27a))\ (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V2y))) \Leftrightarrow \\
& \quad False) \wedge ((p\ (ap\ (ap\ (ap\ (c\_2Eoption\_2EOPTREL\ A\_27a\ A\_27a)\ V0R)\ ( \\
& \quad ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V1x))\ (ap\ (c\_2Eoption\_2ESOME\ A\_27a) \\
& \quad V2y))) \Leftrightarrow (p\ (ap\ (ap\ V0R\ V1x)\ V2y))))))
\end{aligned} \tag{25}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad (ty\_2Eoption\_2Eoption\ A\_27a)\ (ty\_2Eoption\_2Eoption\ A\_27b)) \\
& \quad (ap\ (c\_2Eoption\_2EOPTREL\ A\_27a\ A\_27a)\ V0R))\ (ap\ (c\_2Eoption\_2EOPTION\_MAP \\
& \quad A\_27a\ A\_27b)\ V1abs))\ (ap\ (c\_2Eoption\_2EOPTION\_MAP\ A\_27b\ A\_27a) \\
& \quad V2rep))))))
\end{aligned}$$