

thm_2Equotient_option_2EOPTION_REL_EQ (TMJjBxq1wytdGtq3LV47ogw6HEA7wXP2Vrq)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** *(the* $(\lambda x.x \in A \wedge p)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ *of type* ι .

Definition 5 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27$

Definition 7 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (1)$$

Let $c_2Eoption_2EOPTION_JOIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2EOPTION_JOIN A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Eoption_2Eoption (ty_2Eoption_2Eoption A_27a))}) \quad (2)$$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_MAP A_27a A_27b \in (((ty_2Eoption_2Eoption A_27b)^{(ty_2Eoption_2Eoption A_27a)}(A_27b^{A-27a})) \quad (3)$$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eoption_2Eoption_CASE A_27a A_27b \in (((A_27b^{(A-27b^{A-27a})} A_27b)^{(ty_2Eoption_2Eoption A_27a)}) \quad (4)$$

Let $c_2Eoption_2EIS_NONE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2EIS_NONE A_27a \in (\quad (5)$$

$$2^{(ty_2Eoption_2Eoption A_27a)})$$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2EIS_SOME A_27a \in (\quad (6)$$

$$2^{(ty_2Eoption_2Eoption A_27a)})$$

Definition 8 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2ETHE A_27a \in (A_27a^{(ty_2Eoption_2Eoption A_27a)}) \quad (7)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (8)$$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum \quad (9)$$

$$A0 A1)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum \quad (10)$$

$$A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2})$$

Definition 11 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in \quad (11)$$

$$((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)})$$

Definition 12 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_ABS$

Definition 13 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Definition 14 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS$

Definition 15 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS A_27a) (c$

Definition 16 We define $c_2Eoption_2EOPTREL$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27b})^{A_27a}).\lambda V1x$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{14}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \tag{17}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption\ A_27a).((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a.(V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\
& \quad \forall V0e \in A_27b. (\forall V1f \in (A_27b^{A_27a}). (\forall V2e \in (\\
& \quad \text{ty_2Eoption_2Eoption } A_27a). (\forall V3x \in A_27a. (\forall V4y \in \\
& \quad A_27a. (((\text{ap } (c_2Eoption_2ESOME } A_27a) V3x) = (\text{ap } (c_2Eoption_2ESOME \\
& \quad A_27a) V4y)) \Leftrightarrow (V3x = V4y)))) \wedge ((\forall V5x \in A_27a. ((\text{ap } (c_2Eoption_2ETHE \\
& \quad A_27a) (\text{ap } (c_2Eoption_2ESOME } A_27a) V5x)) = V5x)) \wedge ((\forall V6x \in \\
& \quad A_27a. (\neg((c_2Eoption_2ENONE } A_27a) = (\text{ap } (c_2Eoption_2ESOME \\
& \quad A_27a) V6x)))) \wedge ((\forall V7x \in A_27a. (\neg((\text{ap } (c_2Eoption_2ESOME \\
& \quad A_27a) V7x) = (c_2Eoption_2ENONE } A_27a)))) \wedge ((\forall V8x \in A_27a. \\
& \quad ((\text{p } (\text{ap } (c_2Eoption_2EIS_SOME } A_27a) (\text{ap } (c_2Eoption_2ESOME \\
& \quad A_27a) V8x))) \Leftrightarrow \text{True})) \wedge ((\text{p } (\text{ap } (c_2Eoption_2EIS_SOME } A_27a) \\
& \quad (c_2Eoption_2ENONE } A_27a))) \Leftrightarrow \text{False})) \wedge ((\forall V9x \in (\text{ty_2Eoption_2Eoption} \\
& \quad A_27a). ((\text{p } (\text{ap } (c_2Eoption_2EIS_NONE } A_27a) V9x)) \Leftrightarrow (V9x = (c_2Eoption_2ENONE \\
& \quad A_27a)))) \wedge ((\forall V10x \in (\text{ty_2Eoption_2Eoption } A_27a). ((\neg \\
& \quad (\text{p } (\text{ap } (c_2Eoption_2EIS_SOME } A_27a) V10x))) \Leftrightarrow (V10x = (c_2Eoption_2ENONE \\
& \quad A_27a)))) \wedge ((\forall V11x \in (\text{ty_2Eoption_2Eoption } A_27a). ((\text{p} \\
& \quad (\text{ap } (c_2Eoption_2EIS_SOME } A_27a) V11x)) \Rightarrow ((\text{ap } (c_2Eoption_2ESOME \\
& \quad A_27a) (\text{ap } (c_2Eoption_2ETHE } A_27a) V11x)) = V11x))) \wedge ((\forall V12x \in \\
& \quad (\text{ty_2Eoption_2Eoption } A_27a). ((\text{ap } (\text{ap } (\text{ap } (c_2Eoption_2Eoption_CASE \\
& \quad A_27a) (\text{ty_2Eoption_2Eoption } A_27a)) V12x) (c_2Eoption_2ENONE \\
& \quad A_27a)) (c_2Eoption_2ESOME } A_27a)) = V12x))) \wedge ((\forall V13x \in (\\
& \quad \text{ty_2Eoption_2Eoption } A_27a). ((\text{ap } (\text{ap } (\text{ap } (c_2Eoption_2Eoption_CASE \\
& \quad A_27a) (\text{ty_2Eoption_2Eoption } A_27a)) V13x) V13x) (c_2Eoption_2ESOME \\
& \quad A_27a)) = V13x))) \wedge ((\forall V14x \in (\text{ty_2Eoption_2Eoption } A_27a). \\
& \quad ((\text{p } (\text{ap } (c_2Eoption_2EIS_NONE } A_27a) V14x)) \Rightarrow ((\text{ap } (\text{ap } (\text{ap } (c_2Eoption_2Eoption_CASE \\
& \quad A_27a) A_27b) V14x) V0e) V1f) = V0e))) \wedge ((\forall V15x \in (\text{ty_2Eoption_2Eoption} \\
& \quad A_27a). ((\text{p } (\text{ap } (c_2Eoption_2EIS_SOME } A_27a) V15x)) \Rightarrow ((\text{ap } (\text{ap} \\
& \quad (\text{ap } (c_2Eoption_2Eoption_CASE } A_27a) A_27b) V15x) V0e) V1f) = (\\
& \quad \text{ap } V1f) (\text{ap } (c_2Eoption_2ETHE } A_27a) V15x)))) \wedge ((\forall V16x \in \\
& \quad (\text{ty_2Eoption_2Eoption } A_27a). ((\text{p } (\text{ap } (c_2Eoption_2EIS_SOME \\
& \quad A_27a) V16x)) \Rightarrow ((\text{ap } (\text{ap } (\text{ap } (c_2Eoption_2Eoption_CASE } A_27a) (\\
& \quad \text{ty_2Eoption_2Eoption } A_27a)) V16x) V2e) (c_2Eoption_2ESOME } A_27a)) = \\
& \quad V16x))) \wedge ((\forall V17v \in A_27b. (\forall V18f \in (A_27b^{A_27a}). (\\
& \quad (\text{ap } (\text{ap } (\text{ap } (c_2Eoption_2Eoption_CASE } A_27a) A_27b) (c_2Eoption_2ENONE \\
& \quad A_27a)) V17v) V18f) = V17v))) \wedge ((\forall V19x \in A_27a. (\forall V20v \in \\
& \quad A_27b. (\forall V21f \in (A_27b^{A_27a}). ((\text{ap } (\text{ap } (\text{ap } (c_2Eoption_2Eoption_CASE \\
& \quad A_27a) A_27b) (\text{ap } (c_2Eoption_2ESOME } A_27a) V19x)) V20v) V21f) = \\
& \quad (\text{ap } V21f) V19x)))) \wedge ((\forall V22f \in (A_27b^{A_27a}). (\forall V23x \in \\
& \quad A_27a. ((\text{ap } (\text{ap } (c_2Eoption_2EOPTION_MAP } A_27a) A_27b) V22f) (\\
& \quad \text{ap } (c_2Eoption_2ESOME } A_27a) V23x)) = (\text{ap } (c_2Eoption_2ESOME } A_27b) \\
& \quad (\text{ap } V22f) V23x)))) \wedge ((\forall V24f \in (A_27b^{A_27a}). ((\text{ap } (\text{ap } (c_2Eoption_2EOPTION_MAP \\
& \quad A_27a) A_27b) V24f) (c_2Eoption_2ENONE } A_27a)) = (c_2Eoption_2ENONE \\
& \quad A_27b))) \wedge (((\text{ap } (c_2Eoption_2EOPTION_JOIN } A_27a) (c_2Eoption_2ENONE \\
& \quad (\text{ty_2Eoption_2Eoption } A_27a))) = (c_2Eoption_2ENONE } A_27a)) \wedge \\
& \quad (\forall V25x \in (\text{ty_2Eoption_2Eoption } A_27a). ((\text{ap } (c_2Eoption_2EOPTION_JOIN \\
& \quad A_27a) (\text{ap } (c_2Eoption_2ESOME } (\text{ty_2Eoption_2Eoption } A_27a)) \\
& \quad V25x)) \neq V25x)))))))))
\end{aligned}$$

(19)

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0R \in ((2^{A_{.27a}})^{A_{.27a}}). \\
& (\forall V1x \in A_{.27a}. (\forall V2y \in A_{.27a}. (((p (ap (ap (ap (c_2Eoption_2EOPTREL \\
& A_{.27a} A_{.27a}) V0R) (c_2Eoption_2ENONE A_{.27a})) (c_2Eoption_2ENONE \\
& A_{.27a})) \Leftrightarrow True) \wedge (((p (ap (ap (ap (c_2Eoption_2EOPTREL A_{.27a} A_{.27a}) \\
& V0R) (ap (c_2Eoption_2ESOME A_{.27a}) V1x)) (c_2Eoption_2ENONE A_{.27a})) \Leftrightarrow \\
& False) \wedge (((p (ap (ap (ap (c_2Eoption_2EOPTREL A_{.27a} A_{.27a}) V0R) \\
& (c_2Eoption_2ENONE A_{.27a})) (ap (c_2Eoption_2ESOME A_{.27a}) V2y))) \Leftrightarrow \\
& False) \wedge ((p (ap (ap (ap (c_2Eoption_2EOPTREL A_{.27a} A_{.27a}) V0R) (\\
& ap (c_2Eoption_2ESOME A_{.27a}) V1x)) (ap (c_2Eoption_2ESOME A_{.27a}) \\
& V2y))) \Leftrightarrow (p (ap (ap V0R V1x) V2y))))))))))
\end{aligned} \tag{20}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow ((ap (c_2Eoption_2EOPTREL A_{.27a} \\
& A_{.27a}) (c_2Emin_2E_3D A_{.27a})) = (c_2Emin_2E_3D (ty_2Eoption_2Eoption \\
& A_{.27a}))
\end{aligned}$$