thm_2Equotient__option_2EOPTION__REL__def (TMTjkUG-GpA2mpjZnxbNekuCmwxbMBhqdu9b)

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Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (1)$$

Let $c\_2Eoption\_2EOPTION\_\_JOIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Eoption\_2Eoption\ (ty\_2Eoption\_2Eoption\ A\_27a))}) \quad (2)$$

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \quad (3)$$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (4)$$

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Let $c\_2Eoption\_2EIS\_\_NONE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_NONE\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (5)$$

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $c\_2Eoption\_2EIS\_\_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{6}$$

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{7}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{8}$$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{9}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{10}$$

Definition 8 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS\_$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{11}$$

Definition 9 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_A$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 12 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 13 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 14 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 15 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 16 *We define* c_2Eoption_2EOPTREL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27b})^{A\_27a}).\lambda V1x$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{17}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{20}$$

Assume the following.

$$
\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (
\forall V0e \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).(\forall V2e \in (
ty\_2Eoption\_2Eoption\ A\_27a).((\forall V3x \in A\_27a.(\forall V4y \in
A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}
A\_27a)\ V4y)) \Leftrightarrow (V3x = V4y)))) \wedge ((\forall V5x \in A\_27a.((ap\ (c\_2Eoption\_2ETHE
A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V5x)) = V5x)) \wedge ((\forall V6x \in
A\_27a.(\neg((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}
A\_27a)\ V6x)))) \wedge ((\forall V7x \in A\_27a.(\neg((ap\ (\mathsf{c\_2Eoption\_2ESOME}
A\_27a)\ V7x) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))) \wedge ((\forall V8x \in A\_27a.
((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}
A\_27a)\ V8x))) \Leftrightarrow True)) \wedge (((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)
(\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \Leftrightarrow False) \wedge ((\forall V9x \in (ty\_2Eoption\_2Eoption
A\_27a).((p\ (ap\ (c\_2Eoption\_2EIS\_\_NONE\ A\_27a)\ V9x)) \Leftrightarrow (V9x = (\mathsf{c\_2Eoption\_2ENONE}
A\_27a)))) \wedge ((\forall V10x \in (ty\_2Eoption\_2Eoption\ A\_27a).((\neg
(p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V10x))) \Leftrightarrow (V10x = (\mathsf{c\_2Eoption\_2ENONE}
A\_27a)))) \wedge ((\forall V11x \in (ty\_2Eoption\_2Eoption\ A\_27a).((p
(ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V11x)) \Rightarrow ((ap\ (\mathsf{c\_2Eoption\_2ESOME}
A\_27a)\ (ap\ (c\_2Eoption\_2ETHE\ A\_27a)\ V11x)) = V11x))) \wedge ((\forall V12x \in
(ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE
A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V12x)\ (\mathsf{c\_2Eoption\_2ENONE}
A\_27a))\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)) = V12x)) \wedge ((\forall V13x \in (
ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE
A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V13x)\ V13x)\ (\mathsf{c\_2Eoption\_2ESOME}
A\_27a)) = V13x)) \wedge ((\forall V14x \in (ty\_2Eoption\_2Eoption\ A\_27a).
((p\ (ap\ (c\_2Eoption\_2EIS\_\_NONE\ A\_27a)\ V14x)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE
A\_27a\ A\_27b)\ V14x)\ V0e)\ V1f) = V0e))) \wedge ((\forall V15x \in (ty\_2Eoption\_2Eoption
A\_27a).((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V15x)) \Rightarrow ((ap\ (ap
(ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ V15x)\ V0e)\ V1f) = (
ap\ V1f\ (ap\ (c\_2Eoption\_2ETHE\ A\_27a)\ V15x))))) \wedge ((\forall V16x \in
(ty\_2Eoption\_2Eoption\ A\_27a).((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME
A\_27a)\ V16x)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ (
ty\_2Eoption\_2Eoption\ A\_27a))\ V16x)\ V2e)\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)) =
V16x))) \wedge ((\forall V17v \in A\_27b.(\forall V18f \in (A\_27b^{A\_27a}).(
(ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}
A\_27a))\ V17v)\ V18f) = V17v))) \wedge ((\forall V19x \in A\_27a.(\forall V20v \in
A\_27b.(\forall V21f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE
A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V19x))\ V20v)\ V21f) =
(ap\ V21f\ V19x))))) \wedge ((\forall V22f \in (A\_27b^{A\_27a}).(\forall V23x \in
A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V22f)\ (
ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V23x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)
(ap\ V22f\ V23x))))) \wedge ((\forall V24f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP
A\_27a\ A\_27b)\ V24f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}
A\_27b))) \wedge (((ap\ (c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a)\ (\mathsf{c\_2Eoption\_2ENONE}
(ty\_2Eoption\_2Eoption\ A\_27a))) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \wedge
(\forall V25x \in (ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (c\_2Eoption\_2EOPTION\_\_JOIN
A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Eoption\_2Eoption\ A\_27a))
V25x)) = V25x)))))))))))))))))))))))) \tag{21}
$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTREL}\ A\_27a\ A\_27a)\ V0R)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTREL}\ A\_27a\ A\_27a)\ V0R)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTREL}\ A\_27a\ A\_27a)\ V0R)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2y))) \Leftrightarrow False) \wedge ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTREL}\ A\_27a\ A\_27a)\ V0R)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2y))) \Leftrightarrow (p\ (ap\ (ap\ V0R\ V1x)\ V2y)))))))))$$