

thm_2Equotient__pair_2ECURRY__PRS (TMG8cF8RPM7jsMfCi89AxdvdmFpEGhqKRMV)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 7 We define $c_2Epair_2ECURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27c^{(ty_2Epair$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{3}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{4}$$

Definition 8 We define $c_2\text{Epair_2E_23_23}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f \in (A_27c$

Definition 9 We define $c_2\text{Equotient_2EQUOTIENT}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V$

Definition 10 We define $c_2\text{Equotient_2E_2D_2D_3E}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f$

Assume the following.

$$\text{True} \tag{5}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \text{True})) \tag{6}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\ & \text{nonempty } A_27c \Rightarrow \forall A_27d.\text{nonempty } A_27d \Rightarrow (\forall V0f \in (A_27b^{A_27a}). \\ & (\forall V1g \in (A_27d^{A_27c}). (\forall V2x \in A_27a. (\forall V3y \in \\ & A_27c. ((\text{ap } (\text{ap } (\text{ap } (\text{c_2Epair_2E_23_23 } A_27a } A_27c } A_27b } A_27d) \\ & V0f) V1g) (\text{ap } (\text{ap } (\text{c_2Epair_2E_2C } A_27a } A_27c) V2x) V3y))) = (\text{ap } (\text{ap } \\ & (\text{c_2Epair_2E_2C } A_27b } A_27d) (\text{ap } V0f } V2x)) (\text{ap } V1g } V3y)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\ & \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b^{A_27a}). \\ & (\forall V2rep \in (A_27a^{A_27b}). ((\text{p } (\text{ap } (\text{ap } (\text{ap } (\text{c_2Equotient_2EQUOTIENT} \\ & A_27a } A_27b) V0R) V1abs) V2rep)) \Rightarrow (\forall V3a \in A_27b. ((\text{ap } V1abs \\ & (\text{ap } V2rep } V3a)) = V3a)))))) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\ & \text{nonempty } A_27c \Rightarrow \forall A_27d.\text{nonempty } A_27d \Rightarrow (\forall V0f \in (A_27c^{A_27a}). \\ & (\forall V1g \in (A_27d^{A_27b}). (\forall V2h \in (A_27b^{A_27c}). (\forall V3x \in \\ & A_27a. ((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c_2Equotient_2E_2D_2D_3E } A_27a } A_27b } A_27c \\ & A_27d) V0f) V1g) V2h) V3x) = (\text{ap } V1g (\text{ap } V2h (\text{ap } V0f } V3x)))))) \end{aligned} \tag{9}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow \forall A_27e.nonempty \\
& A_27e \Rightarrow \forall A_27f.nonempty\ A_27f \Rightarrow (\forall V0R1 \in ((2^{A_27a})^{A_27a}). \\
& (\forall V1abs1 \in (A_27d^{A_27a}). (\forall V2rep1 \in (A_27a^{A_27d}). \\
& ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27d)\ V0R1)\ V1abs1) \\
& V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}). (\forall V4abs2 \in (\\
& A_27e^{A_27b}). (\forall V5rep2 \in (A_27b^{A_27e}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& A_27b\ A_27e)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6R3 \in ((2^{A_27c})^{A_27c}). \\
& (\forall V7abs3 \in (A_27f^{A_27c}). (\forall V8rep3 \in (A_27c^{A_27f}). \\
& ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27c\ A_27f)\ V6R3)\ V7abs3) \\
& V8rep3)) \Rightarrow (\forall V9f \in (A_27f^{(ty_2Epair_2Eprod\ A_27d\ A_27e)}). \\
& (\forall V10a \in A_27d. (\forall V11b \in A_27e. ((ap\ (ap\ (ap\ (c_2Epair_2ECURRY \\
& A_27d\ A_27e\ A_27f)\ V9f)\ V10a)\ V11b) = (ap\ V7abs3\ (ap\ (ap\ (ap\ (c_2Epair_2ECURRY \\
& A_27a\ A_27b\ A_27c)\ (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ (ty_2Epair_2Eprod \\
& A_27a\ A_27b)\ A_27f\ (ty_2Epair_2Eprod\ A_27d\ A_27e)\ A_27c)\ (ap\ (ap \\
& (c_2Epair_2E_23_23\ A_27a\ A_27b\ A_27d\ A_27e)\ V1abs1)\ V4abs2))\ V8rep3) \\
& V9f))\ (ap\ V2rep1\ V10a)\ (ap\ V5rep2\ V11b))))))))))))))
\end{aligned}$$