thm_2Equotient__pair_2EPAIR__MAP__PRS (TMN8KBhNLCfdHZzJ9BfE4ANE5wHJWLMdGNj)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 8 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{3}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{4}$$

Definition 9 *We define* c_2Epair_2E_23_23 *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f \in (A\_27c$

Definition 10 *We define* c_2Equotient_2EQUOTIENT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

Definition 11 *We define* c_2Equotient_2E_2D_2D_3E *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q \in A\_27a.(\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1q)\ V2r))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27d^{A\_27c}).(\forall V2x \in A\_27a.(\forall V3y \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_23\_23}\ A\_27a\ A\_27c\ A\_27b\ A\_27d)\ V0f)\ V1g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27c)\ V2x)\ V3y)) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27d)\ (ap\ V0f\ V2x))\ (ap\ V1g\ V3y))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}).(\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3a \in A\_27b.((ap\ V1abs\ (ap\ V2rep\ V3a)) = V3a)))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0f \in (A\_27c^{A\_27a}). (\forall V1g \in (A\_27d^{A\_27b}).(\forall V2h \in (A\_27b^{A\_27c}).(\forall V3x \in A\_27a.((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27a\ A\_27b\ A\_27c\ A\_27d)\ V0f)\ V1g)\ V2h)\ V3x) = (ap\ V1g\ (ap\ V2h\ (ap\ V0f\ V3x))))))))) \tag{11}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow \forall A\_27e.nonempty\ A\_27e \Rightarrow \forall A\_27f.nonempty\ A\_27f \Rightarrow \forall A\_27g.nonempty\ A\_27g \Rightarrow \forall A\_27h.nonempty\ A\_27h \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}). (\forall V1abs1 \in (A\_27e^{A\_27a}).(\forall V2rep1 \in (A\_27a^{A\_27e}). ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27e)\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in (A\_27f^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27f}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27b\ A\_27f)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6R3 \in ((2^{A\_27c})^{A\_27c}). (\forall V7abs3 \in (A\_27g^{A\_27c}).(\forall V8rep3 \in (A\_27c^{A\_27g}). ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27c\ A\_27g)\ V6R3)\ V7abs3)\ V8rep3)) \Rightarrow (\forall V9R4 \in ((2^{A\_27d})^{A\_27d}).(\forall V10abs4 \in (A\_27h^{A\_27d}).(\forall V11rep4 \in (A\_27d^{A\_27h}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27d\ A\_27h)\ V9R4)\ V10abs4)\ V11rep4)) \Rightarrow (\forall V12f \in (A\_27f^{A\_27e}).(\forall V13g \in (A\_27h^{A\_27g}).((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_23\_23}\ A\_27e\ A\_27g\ A\_27f\ A\_27h)\ V12f)\ V13g) = (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ (ty\_2Epair\_2Eprod\ A\_27e\ A\_27g)\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27d)\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c)\ (ty\_2Epair\_2Eprod\ A\_27f\ A\_27h))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_23\_23}\ A\_27e\ A\_27g\ A\_27a\ A\_27c)\ V2rep1)\ V8rep3))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_23\_23}\ A\_27b\ A\_27d\ A\_27f\ A\_27h)\ V4abs2)\ V10abs4))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_23\_23}\ A\_27a\ A\_27c\ A\_27b\ A\_27d)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27a\ A\_27f\ A\_27e\ A\_27b)\ V1abs1)\ V5rep2)\ V12f))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27c\ A\_27h\ A\_27g\ A\_27d)\ V7abs3)\ V11rep4)\ V13g))))))))))))))))))))))$$