

# thm\_2Equotient\_\_pair\_2EPAIR\_\_REL\_\_THM (TMZYJk6owThg671pLhcKi9DUSZQ1bos6cd)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow Q)$  of type  $\iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 6** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Ebool\_2E\_2F\_5C$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND A\_27a A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \tag{3}$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \tag{4}$$

**Definition 7** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 8** We define  $c\_2Equotient\_pair\_2E\_23\_23\_23$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0R1 \in ((A\_27c^{A\_27b})^{A\_27a})$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & \quad nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}). (\forall V1x \in \\ & \quad A\_27a. (\forall V2y \in A\_27b. ((ap\ (ap\ (c\_2Epair\_2EUNCURRY\ A\_27a \\ & \quad A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V1x)\ V2y))) = \\ & \quad (ap\ (ap\ V0f\ V1x)\ V2y)))))) \end{aligned} \tag{5}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & \quad nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\ & \quad (2^{A\_27c})^{A\_27a}). (\forall V1R2 \in ((2^{A\_27d})^{A\_27b}). (\forall V2a \in \\ & \quad A\_27a. (\forall V3b \in A\_27b. (\forall V4c \in A\_27c. (\forall V5d \in A\_27d. \\ & \quad ((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_pair\_2E\_23\_23\_23\ A\_27a\ A\_27b \\ & \quad A\_27c\ A\_27d)\ V0R1)\ V1R2)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2a) \\ & \quad V3b))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27c\ A\_27d)\ V4c)\ V5d))) \Leftrightarrow ((p\ (ap\ ( \\ & \quad ap\ V0R1\ V2a)\ V4c)) \wedge (p\ (ap\ (ap\ V1R2\ V3b)\ V5d)))))))))) \end{aligned}$$