

thm_2Equotient__pair_2EUNCURRY__RSP (TMTSDs9AuXLvKz9VYXVzdCy2Wid8jvP6ZUX)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 3 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{27a}})) (\lambda V1t \in 2.V1t)) (\lambda V2t \in 2.V2t))$

Definition 5 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 6 We define `c_2Equotient_2EQUOTIENT` to be $\lambda A_{27a} : \iota.\lambda A_{27b} : \iota.\lambda V0R \in ((2^{A_{27a}})^{A_{27b}}).\lambda V1P \in (2^{A_{27a}}).$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2E_3F` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap V0P (ap (c_2Emin_2E_40 A_{27a} P))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let `c_2Epair_2ESND` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow c_2Epair_2ESND A_{27a} A_{27b} \in (A_{27b}^{(ty_2Epair_2Eprod A_{27a} A_{27b})}) \tag{2}$$

Let `c_2Epair_2EFST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow c_2Epair_2EFST A_{27a} A_{27b} \in (A_{27a}^{(ty_2Epair_2Eprod A_{27a} A_{27b})}) \tag{3}$$

Definition 9 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27b})$

Definition 10 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R1 \in ((2^{A_27a})^{A_27b})$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (4)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2E_2C\ A_27a\ A_27b\ V0x\ V1y))$

Definition 12 We define $c_2Equotient_pair_2E_23_23_23$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0R1 \in ((2^{A_27a})^{A_27b})^{A_27c}$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b).(\exists V1q \in A_27a. \\ (\exists V2r \in A_27b.(V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b) \\ V1q)\ V2r)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c)^{A_27b})^{A_27a}).(\forall V1x \in \\ A_27a.(\forall V2y \in A_27b.((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\ A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))) = \\ (ap\ (ap\ V0f\ V1x)\ V2y)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\ (2^{A_27c})^{A_27a}).(\forall V1R2 \in ((2^{A_27d})^{A_27b}).(\forall V2a \in \\ A_27a.(\forall V3b \in A_27b.(\forall V4c \in A_27c.(\forall V5d \in A_27d. \\ ((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_pair_2E_23_23_23\ A_27a\ A_27b \\ A_27c\ A_27d)\ V0R1)\ V1R2)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a) \\ V3b))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27c\ A_27d)\ V4c)\ V5d)))) \Leftrightarrow ((p\ (ap\ (\\ ap\ V0R1\ V2a)\ V4c)) \wedge (p\ (ap\ (ap\ V1R2\ V3b)\ V5d)))))))))) \end{aligned} \quad (7)$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow \forall A_27e.nonempty \\
& A_27e \Rightarrow \forall A_27f.nonempty\ A_27f \Rightarrow (\forall V0R1 \in ((2^{A_27a})^{A_27a}). \\
& (\forall V1abs1 \in (A_27d^{A_27a}). (\forall V2rep1 \in (A_27a^{A_27d}). \\
& ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27d)\ V0R1)\ V1abs1) \\
& V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}). (\forall V4abs2 \in (\\
& A_27e^{A_27b}). (\forall V5rep2 \in (A_27b^{A_27e}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& A_27b\ A_27e)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6R3 \in ((2^{A_27c})^{A_27c}). \\
& (\forall V7abs3 \in (A_27f^{A_27c}). (\forall V8rep3 \in (A_27c^{A_27f}). \\
& ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27c\ A_27f)\ V6R3)\ V7abs3) \\
& V8rep3)) \Rightarrow (\forall V9f1 \in ((A_27c^{A_27b})^{A_27a}). (\forall V10f2 \in \\
& ((A_27c^{A_27b})^{A_27a}). ((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\
& A_27a\ (A_27c^{A_27b}))\ V0R1)\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\
& A_27b\ A_27c)\ V3R2)\ V6R3))\ V9f1)\ V10f2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\
& (ty_2Epair_2Eprod\ A_27a\ A_27b)\ A_27c)\ (ap\ (ap\ (c_2Equotient_pair_2E_23_23_23 \\
& A_27a\ A_27b\ A_27a\ A_27b)\ V0R1)\ V3R2))\ V6R3)\ (ap\ (c_2Epair_2EUNCURRY \\
& A_27a\ A_27b\ A_27c)\ V9f1))\ (ap\ (c_2Epair_2EUNCURRY\ A_27a\ A_27b\ A_27c) \\
& V10f2))))))))))))))
\end{aligned}$$