thm_2Equotient__pred__set_2EFINITER__INDUCT (TMZrDGeKBje3s7V3j996Pocc8fZ8J5Q5zVu)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 10 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 11 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{3}$$

Definition 12 *We define* $\mathsf{c\_2Equotient\_\_pred\_\_set\_2EINSERTR}$ *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V$

Definition 13 *We define* $\mathsf{c\_2Ecombin\_2EW}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x$

Definition 14 *We define* $\mathsf{c\_2Equotient\_2Erespects}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\mathsf{c\_2Ecombin\_2EW}\ A\_27a\ A\_2$

Definition 15 *We define* $\mathsf{c\_2Ebool\_2ERES\_\_FORALL}$ *to be* $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).($

Definition 16 *We define* $\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27}$

Definition 17 *We define* $\mathsf{c\_2Equotient\_\_pred\_\_set\_2EFINITER}$ *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))) \Leftrightarrow (p\ (ap\ (ap\ V0R\ V1x)\ V1x))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1s1 \in (2^{A\_27a}).(\forall V2s2 \in (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ 2)\ V0R)\ (\mathsf{c\_2Emin\_2E\_3D}\ 2))\ V1s1)\ V2s2)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Equotient\_\_pred\_\_set\_2EFINITER}\ A\_27a)\ V0R)\ V1s1)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Equotient\_\_pred\_\_set\_2EFINITER}\ A\_27a)\ V0R)\ V2s2))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (p\ (ap\ (ap\ (\mathsf{c\_2Equotient\_\_pred\_\_set\_2EFINITER}\ A\_27a)\ V0R)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ (2^{A\_27a}))\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ (2^{A\_27a})\ 2)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ 2)\ V0R)\ (\mathsf{c\_2Emin\_2E\_3D}\ 2))))\ (\lambda V1s \in (2^{A\_27a}).(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ (ap\ (\mathsf{c\_2Equotient\_\_pred\_\_set\_2EFINITER}\ A\_27a)\ V0R)\ V1s))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ (\lambda V2x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Equotient\_\_pred\_\_set\_2EFINITER}\ A\_27a)\ V0R)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_\_pred\_\_set\_2EINSERTR}\ A\_27a)\ V0R)\ V2x)\ V1s))))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ (\lambda V1x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ (2^{A\_27a}))\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ (2^{A\_27a})\ 2)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ 2)\ V0R)\ (\mathsf{c\_2Emin\_2E\_3D}\ 2))))\ (\lambda V2s \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ 2)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ V2s))\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ 2)\ V0R)\ (\mathsf{c\_2Emin\_2E\_3D}\ 2))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_\_pred\_\_set\_2EINSERTR}\ A\_27a)\ V0R)\ V1x)\ V2s))\ V2s))))))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1f \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ V0P)\ V1f)) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0P)) \Rightarrow (p\ (ap\ V1f\ V2x))))))) \tag{19}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ (2^{(2^{A\_27a})}))\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ (2^{(2^{A\_27a})})\ 2)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ (2^{A\_27a})\ 2)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ 2)\ V0R)\ (\mathsf{c\_2Emin\_2E\_3D}\ 2)))\ (\mathsf{c\_2Emin\_2E\_3D}\ 2))))\ (\lambda V1P \in (2^{(2^{A\_27a})}).(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ V1P\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ (2^{A\_27a}))\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ (2^{A\_27a})\ 2)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ 2)\ V0R)\ (\mathsf{c\_2Emin\_2E\_3D}\ 2))))\ (\lambda V2s \in (2^{A\_27a}).(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Equotient\_\_pred\_\_set\_2EFINITER}\ A\_27a)\ V0R)\ V2s))\ (ap\ V1P\ V2s)))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ (\lambda V3e \in A\_27a.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3e)\ V2s)))\ (ap\ V1P\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_\_pred\_\_set\_2EINSERTR}\ A\_27a)\ V0R)\ V3e)\ V2s))))))))))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ (2^{A\_27a}))\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ (2^{A\_27a})\ 2)\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ 2)\ V0R)\ (\mathsf{c\_2Emin\_2E\_3D}\ 2))))\ (\lambda V4s \in (2^{A\_27a}).(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ (ap\ (\mathsf{c\_2Equotient\_\_pred\_\_set\_2EFINITER}\ A\_27a)\ V0R)\ V4s))\ (ap\ V1P\ V4s)))))))))$$