

thm_2Equotient__pred__set_2EGSPEC__PRS (TMHxQa8iF2rbHC7AE7Jaz19UDfb9z95T6Uh)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)) P))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2ESND A.27a A.27b \in (A.27b^{(ty_2Epair_2Eprod A.27a A.27b)}) \tag{2}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2EFST A.27a A.27b \in (A.27a^{(ty_2Epair_2Eprod A.27a A.27b)}) \tag{3}$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)) V0t1 t2))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2EABS_prod A.27a A.27b \in ((ty_2Epair_2Eprod A.27a A.27b)^{(2^{A-27b})^{A-27a}}) \tag{4}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 9 We define $c_2Epair_2E_23_23$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f \in (A_27c$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (5)$$

Definition 10 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda$

Definition 11 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x$

Definition 12 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 13 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A$

Definition 14 We define $c_2Equotient_2E_2D_2D_23E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f$

Definition 15 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 16 We define $c_2Equotient_pair_2E_23_23_23$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda$

Definition 17 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in ((A_27b^{A_27a})^{A_27a}).(\lambda V1x$

Definition 18 We define $c_2Equotient_2Erespects$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(c_2Ecombin_2EW\ A_27a\ A_27b$

Definition 19 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Definition 20 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the (\lambda x.x \in A)\lambda$
of type $\iota \Rightarrow \iota$.

Definition 21 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P (ap (c_2Emin_2E_40$

Definition 22 We define $c_2Ebool_2ERES_EXISTS$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).($

Definition 23 We define $c_2Equotient_pred_set_2EGSPECR$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R1 \in ((2^{A_27a})^{A_27a})$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). ((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V0x))\ (ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ V0x)) = V0x)) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V1y))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0f \in ((ty.2Epair.2Eprod\ A.27a\ 2)^{A.27b}).(\forall V1v \in \\ & A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1v)\ (ap\ (c.2Epred_set.2EGSPEC \\ & \quad A.27a\ A.27b)\ V0f))) \Leftrightarrow (\exists V2x \in A.27b.((ap\ (ap\ (c.2Epair.2E2C \\ & \quad A.27a\ 2)\ V1v)\ c.2Ebool.2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0R \in ((2^{A.27a})^{A.27a}).(\forall V1abs \in (A.27b^{A.27a}). \\ & (\forall V2rep \in (A.27a^{A.27b}).((p\ (ap\ (ap\ (ap\ (c.2Equotient.2EQUOTIENT \\ & \quad A.27a\ A.27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3a \in A.27b.((ap\ V1abs \\ & \quad (ap\ V2rep\ V3a)) = V3a)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0R \in ((2^{A.27a})^{A.27a}).(\forall V1abs \in (A.27b^{A.27a}). \\ & (\forall V2rep \in (A.27a^{A.27b}).((p\ (ap\ (ap\ (ap\ (c.2Equotient.2EQUOTIENT \\ & \quad A.27a\ A.27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3a \in A.27b.(p\ (ap\ (ap \\ & \quad V0R\ (ap\ V2rep\ V3a))\ (ap\ V2rep\ V3a)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0R \in ((2^{A.27a})^{A.27a}).(\forall V1abs \in (A.27b^{A.27a}). \\ & (\forall V2rep \in (A.27a^{A.27b}).((p\ (ap\ (ap\ (ap\ (c.2Equotient.2EQUOTIENT \\ & \quad A.27a\ A.27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3a \in A.27b.(\forall V4b \in \\ & A.27b.((p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3a))\ (ap\ V2rep\ V4b))) \Leftrightarrow (V3a = V4b)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0f \in (A.27c^{A.27a}). \\ & \quad (\forall V1g \in (A.27d^{A.27b}).(\forall V2h \in (A.27b^{A.27c}).(\forall V3x \in \\ & A.27a.((ap\ (ap\ (ap\ (ap\ (c.2Equotient.2E.2D.2D.3E\ A.27a\ A.27b\ A.27c \\ & \quad A.27d)\ V0f)\ V1g)\ V2h)\ V3x) = (ap\ V1g\ (ap\ V2h\ (ap\ V0f\ V3x)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0R \in ((2^{A.27a})^{A.27a}). \\ & \quad (\forall V1x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ (ap\ (\\ & c.2Equotient.2Erespects\ A.27a\ 2)\ V0R))) \Leftrightarrow (p\ (ap\ (ap\ V0R\ V1x)\ V1x)))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27c})^{A_27a}).(\forall V1R2 \in ((2^{A_27d})^{A_27b}).(\forall V2a \in \\
& \quad A_27a.(\forall V3b \in A_27b.(\forall V4c \in A_27c.(\forall V5d \in A_27d. \\
& \quad ((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_pair_2E_23_23_23\ A_27a\ A_27b \\
& \quad A_27c\ A_27d)\ V0R1)\ V1R2)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a) \\
& \quad V3b))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27c\ A_27d)\ V4c)\ V5d))) \Leftrightarrow ((p\ (ap\ (\\
& \quad ap\ V0R1\ V2a)\ V4c)) \wedge (p\ (ap\ (ap\ V1R2\ V3b)\ V5d)))))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b)^{A_27a}).(\forall V1s \in (2^{A_27b}).(\forall V2x \in \\
& \quad A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27a\ 2\ A_27b\ 2)\ V0f)\ (c_2Ecombin_2EI\ 2))\ V1s))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27b)\ (ap\ V0f\ V2x))\ V1s))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R1 \in ((2^{A_27b})^{A_27a}).(\forall V1R2 \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V2f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}).(\forall V3v \in \\
& \quad A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3v)\ (ap\ (ap\ (ap\ (c_2Equotient_pred_set_2EGSPECR \\
& \quad A_27b\ A_27a)\ V0R1)\ V1R2)\ V2f))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_EXISTS \\
& \quad A_27b)\ (ap\ (c_2Equotient_2Erespects\ A_27b\ 2)\ V0R1))\ (\lambda V4x \in \\
& \quad A_27b.(ap\ (ap\ (ap\ (ap\ (c_2Equotient_pair_2E_23_23_23\ A_27a\ 2 \\
& \quad A_27a\ 2)\ V1R2)\ (c_2Emin_2E_3D\ 2))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a \\
& \quad 2)\ V3v)\ c_2Ebool_2ET))\ (ap\ V2f\ V4x)))))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1f \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (c_2Ebool_2ERES_EXISTS\ A_27a)\ V0P)\ V1f))) \Leftrightarrow \\
& \quad (\exists V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0P)) \wedge \\
& \quad (p\ (ap\ V1f\ V2x))))))
\end{aligned} \tag{27}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\ & \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\ & \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\ & \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\ & \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in ((ty_2Epair_2Eprod \\ & \quad A_27d\ 2)^{A_27c}).((ap\ (c_2Epred_set_2EGSPEC\ A_27d\ A_27c)\ V6f) = \\ & \quad (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27d\ 2\ A_27b\ 2)\ V5rep2) \\ & \quad (c_2Ecombin_2EI\ 2))\ (ap\ (ap\ (ap\ (c_2Equotient_pred_set_2EGSPECR \\ & \quad A_27a\ A_27b)\ V0R1)\ V3R2)\ (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\ & \quad A_27a\ (ty_2Epair_2Eprod\ A_27d\ 2)\ A_27c\ (ty_2Epair_2Eprod\ A_27b \\ & \quad 2))\ V1abs1)\ (ap\ (ap\ (c_2Epair_2E_23_23\ A_27d\ 2\ A_27b\ 2)\ V5rep2) \\ & \quad (c_2Ecombin_2EI\ 2))))))))))))))))) \end{aligned}$$