

thm_2Equotient__pred__set_2EINSERT__PRS (TMSqZgEiq3D7taxvK1XJD3h44iZPr3Tuxmz)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).)(ap V1f V0x))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).)(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2) (c_2Epair_2EABS_prod A_27a A_27b))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \tag{3}$$

Definition 9 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 2) (c_2Epair_2EABS_prod A_27a A_27a))$

Definition 10 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda$

Definition 11 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 12 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c)^{A_27b})^{A_27a}).\lambda$

Definition 13 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a)^{A_27a}) A$

Definition 14 We define $c_2Equotient_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f$

Definition 15 We define $c_2Equotient_pred_set_2EINSERTR$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{5}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\ & (2^{A_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN \\ & A_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V1t))))))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.(\forall V2s \in (2^{A_27a}).((p (ap (ap (c_2Ebool_2EIN A_27a) \\ & V0x) (ap (ap (c_2Epred_set_2EINSERT A_27a) V1y) V2s))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) V2s))))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b)^{A_27a}). \\ & (\forall V2rep \in (A_27a)^{A_27b}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\ & A_27a A_27b) V0R) V1abs) V2rep)) \Rightarrow (\forall V3a \in A_27b.((ap V1abs \\ & (ap V2rep V3a)) = V3a)))))) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b)^{A_27a}). \\ & (\forall V2rep \in (A_27a)^{A_27b}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\ & A_27a A_27b) V0R) V1abs) V2rep)) \Rightarrow (\forall V3a \in A_27b.(\forall V4b \in \\ & A_27b.((p (ap (ap V0R (ap V2rep V3a)) (ap V2rep V4b))) \Leftrightarrow (V3a = V4b)))))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}).(\forall V1s \in (2^{A_27b}).(\forall V2x \in \\ A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\ A_27a\ 2\ A_27b\ 2)\ V0f)\ (c_2Ecombin_2EI\ 2))\ V1s)))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A_27b)\ (ap\ V0f\ V2x))\ V1s)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\ & \quad (\forall V1x \in A_27a.(\forall V2s \in (2^{A_27a}).(\forall V3y \in A_27a. \\ ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3y)\ (ap\ (ap\ (ap\ (c_2Equotient_pred_set_2EINSERTR \\ A_27a)\ V0R)\ V1x)\ V2s)))) \Leftrightarrow ((p\ (ap\ (ap\ V0R\ V3y)\ V1x)) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A_27a)\ V3y)\ V2s)))))) \end{aligned} \quad (11)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\ (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3s \in (2^{A_27b}).(\forall V4x \in \\ A_27b.((ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27b)\ V4x)\ V3s) = (ap\ (\\ ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27b\ 2\ A_27a\ 2)\ V2rep)\ (c_2Ecombin_2EI \\ 2))\ (ap\ (ap\ (ap\ (c_2Equotient_pred_set_2EINSERTR\ A_27a)\ V0R) \\ (ap\ V2rep\ V4x))\ (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b \\ 2)\ V1abs)\ (c_2Ecombin_2EI\ 2))\ V3s)))))) \end{aligned}$$