

# thm\_2Equotient\_\_pred\_\_set\_2EINTER\_\_PRS (TM- RDRQrvJqGBQMVRpZCa4gxTPrPQRjk9vzL)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (\lambda V1P \in (2^{A-27a}).V1P)) (\lambda V2P \in (2^{A-27a}).V2P)))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 6** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A-27b})^{A-27a}}) \quad (2)$$

**Definition 7** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod A\_27a A\_27b) (ty\_2Epair\_2Eprod A\_27a A\_27b))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A-27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A-27b}}) \quad (3)$$

**Definition 8** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c\_2Epair\_2E\_2C A\_27a A\_27a) (ty\_2Epair\_2Eprod A\_27a A\_27a))$

**Definition 9** We define  $c\_2Equotient\_2EQUOTIENT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1L \in ((2^{A-27b})^{A-27b}).(ap (c\_2Epair\_2E\_2C A\_27a A\_27b) (ty\_2Epair\_2Eprod A\_27a A\_27b))$

**Definition 10** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x$

**Definition 11** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 12** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A$

**Definition 13** We define  $c\_2Equotient\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{5}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\ & (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN \\ & A\_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V1t))))))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\ & (2^{A\_27a}).(\forall V2x \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN A\_27a) \\ & V2x) (ap (ap (c\_2Epred\_set\_2EINTER A\_27a) V0s) V1t))) \Leftrightarrow (p (ap \\ & (ap (c\_2Ebool\_2EIN A\_27a) V2x) V0s)) \wedge (p (ap (ap (c\_2Ebool\_2EIN \\ & A\_27a) V2x) V1t))))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & \forall V0R \in ((2^{A\_27a})^{A\_27b}).(\forall V1abs \in (A\_27b^{A\_27a}). \\ & (\forall V2rep \in (A\_27a^{A\_27b}).((p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT \\ & A\_27a A\_27b) V0R) V1abs) V2rep)) \Rightarrow (\forall V3a \in A\_27b.((ap V1abs \\ & (ap V2rep V3a)) = V3a)))))) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & \forall V0f \in (A\_27b^{A\_27a}).(\forall V1s \in (2^{A\_27b}).(\forall V2x \in \\ & A\_27a.((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) (ap (ap (ap (c\_2Equotient\_2E\_2D\_2D\_3E \\ & A\_27a 2 A\_27b 2) V0f) (c\_2Ecombin\_2EI 2) V1s))) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN \\ & A\_27b) (ap V0f V2x) V1s)))))) \end{aligned} \tag{9}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\ & (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ & \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3s \in (2^{A\_27b}).(\forall V4t \in \\ & \quad (2^{A\_27b}).((ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27b)\ V3s)\ V4t) = ( \\ & \quad ap\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27b\ 2\ A\_27a\ 2)\ V2rep)\ ( \\ & \quad \quad c\_2Ecombin\_2EI\ 2))\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27a)\ (ap \\ & \quad (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27a\ 2\ A\_27b\ 2)\ V1abs)\ (c\_2Ecombin\_2EI \\ & \quad \quad 2))\ V3s))\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27a\ 2\ A\_27b \\ & \quad \quad 2)\ V1abs)\ (c\_2Ecombin\_2EI\ 2))\ V4t)))))))))) \end{aligned}$$