

thm_2Equotient__pred__set_2EINTER__RSP
(TMKrAXD-
Kmqm8k1KAUhe6uT99m5yuwV3gKbv)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \tag{3}$$

Definition 8 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epair_2EABS_prod$

Definition 9 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R1 \in ((2^{A_27a})^{A_27a})$

Definition 10 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a})$.

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{5}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{6}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{7}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1s \in (2^{A_27a}). (\forall V2t \in (2^{A_27a}). ((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a\ 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V1s)\ V2t)) \Leftrightarrow (\forall V3x \in A_27a. (\forall V4y \in A_27a. ((p\ (ap\ (ap\ V0R\ V3x)\ V4y)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V4y)\ V2t)))))))))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b)^{A_27a}). (\forall V2rep \in (A_27a)^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3s \in (2^{A_27a}). (\forall V4t \in (2^{A_27a}). (\forall V5x \in A_27a. (\forall V6y \in A_27a. (((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a\ 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V3s)\ V4t)) \wedge (p\ (ap\ (ap\ V0R\ V5x)\ V6y))) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V5x)\ V3s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V6y)\ V4t)))))))))) \end{aligned} \tag{10}$$

Theorem 1

$$\begin{aligned}
& \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_{27a}})^{A_{27a}}).(\forall V1abs \in (A_{27b}^{A_{27a}}). \\
& (\forall V2rep \in (A_{27a}^{A_{27b}}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_{27a}\ A_{27b})\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3s1 \in (2^{A_{27a}}).(\forall V4s2 \in \\
& \quad (2^{A_{27a}}).(\forall V5t1 \in (2^{A_{27a}}).(\forall V6t2 \in (2^{A_{27a}}). \\
& \quad (((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_{27a}\ 2)\ V0R) \\
& (c_2Emin_2E_3D\ 2))\ V3s1)\ V4s2)) \wedge (p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\
& \quad A_{27a}\ 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V5t1)\ V6t2))) \Rightarrow (p\ (ap\ (ap\ (ap\ (\\
& \quad ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_{27a}\ 2)\ V0R)\ (c_2Emin_2E_3D \\
& \quad 2))\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_{27a})\ V3s1)\ V5t1))\ (ap\ (ap \\
& \quad (c_2Epred_set_2EINTER\ A_{27a})\ V4s2)\ V6t2))))))))))
\end{aligned}$$