thm_2Equotient__pred__set_2EIN__SET__MAP (TMRi7DUmkaZYs9QA5eRM5e7iYaqqtHFjT2s)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 4 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 5 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 6 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 7 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 8 *We define* c_2Equotient_2E_2D_2D_3E *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ V0P)) \Leftrightarrow (p\ (ap\ V0P\ V1x))))) \tag{4}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d. nonempty\ A\_27d \Rightarrow (\forall V0f \in (A\_27c^{A\_27a}). (\forall V1g \in (A\_27d^{A\_27b}). (\forall V2h \in (A\_27b^{A\_27c}). (\forall V3x \in A\_27a. ((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27a\ A\_27b\ A\_27c\ A\_27d)\ V0f)\ V1g)\ V2h)\ V3x) = (ap\ V1g\ (ap\ V2h\ (ap\ V0f\ V3x))))))))) \tag{5}$$

Theorem 1

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1s \in (2^{A\_27b}). (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27a\ 2\ A\_27b\ 2)\ V0f)\ (\mathsf{c\_2Ecombin\_2EI}\ 2))\ V1s))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V0f\ V2x))\ V1s))))))$$