

thm_2Equotient__pred__set_2EPSUBSETR__RSP
(TMWYAmGZEpgjLV-
GovMpYGa6mAdhD4NFmdQY)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 6 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 7 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a$

Definition 8 We define $c_2Equotient_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$
of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 11 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27b}).\lambda$

Definition 12 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R1 \in ((2^{A_27a})^{A_27b})$

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F_5C$

Definition 14 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Definition 15 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in ((A_27b^{A_27a})^{A_27a}).(\lambda V1x \in A_27a$

Definition 16 We define $c_2Equotient_2Erespects$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(c_2Ecombin_2EW A_27a A_27b$

Definition 17 We define `c_2Ebool_2ERES_FORALL` to be $\lambda A_{.27a} : \iota. (\lambda V0p \in (2^{A_{.27a}}). (\lambda V1m \in (2^{A_{.27a}}).$

Definition 18 We define `c_2Equotient_pred_set_2ESUBSETR` to be $\lambda A_{.27a} : \iota. \lambda V0R \in ((2^{A_{.27a}})^{A_{.27a}}). \lambda V$

Definition 19 We define `c_2Equotient_pred_set_2EPSUBSETR` to be $\lambda A_{.27a} : \iota. \lambda V0R \in ((2^{A_{.27a}})^{A_{.27a}}). \lambda$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\ & (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p \ V0t)))))) \end{aligned} \tag{4}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. nonempty \ A_{.27a} \Rightarrow (p \ (ap \ (ap \ (ap \ (c_2Equotient_2EQUOTIENT \\ & \ A_{.27a} \ A_{.27a}) \ (c_2Emin_2E_3D \ A_{.27a}) \ (c_2Ecombin_2EI \ A_{.27a})) \ (\\ & \ c_2Ecombin_2EI \ A_{.27a}))) \end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}. nonempty \ A_{.27b} \Rightarrow \forall A_{.27c}. \\ & nonempty \ A_{.27c} \Rightarrow \forall A_{.27d}. nonempty \ A_{.27d} \Rightarrow (\forall V0R1 \in (\\ & (2^{A_{.27a}})^{A_{.27a}}). (\forall V1abs1 \in (A_{.27c}^{A_{.27a}}). (\forall V2rep1 \in \\ & (A_{.27a}^{A_{.27c}}). ((p \ (ap \ (ap \ (ap \ (c_2Equotient_2EQUOTIENT \ A_{.27a} \ A_{.27c}) \\ & \ V0R1) \ V1abs1) \ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_{.27b}})^{A_{.27b}}). (\forall V4abs2 \in \\ & (A_{.27d}^{A_{.27b}}). (\forall V5rep2 \in (A_{.27b}^{A_{.27d}}). ((p \ (ap \ (ap \ (ap \ (c_2Equotient_2EQUOTIENT \\ & \ A_{.27b} \ A_{.27d}) \ V3R2) \ V4abs2) \ V5rep2)) \Rightarrow (p \ (ap \ (ap \ (ap \ (c_2Equotient_2EQUOTIENT \\ & \ (A_{.27b}^{A_{.27a}}) \ (A_{.27d}^{A_{.27c}})) \ (ap \ (ap \ (c_2Equotient_2E_3D_3D_3E \\ & \ A_{.27a} \ A_{.27b}) \ V0R1) \ V3R2)) \ (ap \ (ap \ (c_2Equotient_2E_2D_2D_3E \ A_{.27c} \\ & \ A_{.27b} \ A_{.27a} \ A_{.27d}) \ V2rep1) \ V4abs2)) \ (ap \ (ap \ (c_2Equotient_2E_2D_2D_3E \\ & \ A_{.27a} \ A_{.27d} \ A_{.27c} \ A_{.27b}) \ V1abs1) \ V5rep2)))))))))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& \quad A_27a.(\forall V5y1 \in A_27a.(\forall V6y2 \in A_27a.(((p\ (ap\ (ap\ V0R \\
& \quad V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2))))))))))))) \\
& \hspace{15em} (7)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3s1 \in (2^{A_27a}).(\forall V4s2 \in \\
& \quad (2^{A_27a}).(\forall V5t1 \in (2^{A_27a}).(\forall V6t2 \in (2^{A_27a}). \\
& \quad (((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a\ 2)\ V0R) \\
& \quad (c_2Emin_2E_3D\ 2))\ V3s1)\ V4s2)) \wedge (p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\
& \quad A_27a\ 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V5t1)\ V6t2))) \Rightarrow ((p\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_pred_set_2ESUBSETR\ A_27a)\ V0R)\ V3s1)\ V5t1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (ap\ (c_2Equotient_pred_set_2ESUBSETR\ A_27a)\ V0R) \\
& \quad V4s2)\ V6t2))))))))))))) \\
& \hspace{15em} (8)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3s1 \in (2^{A_27a}).(\forall V4s2 \in \\
& \quad (2^{A_27a}).(\forall V5t1 \in (2^{A_27a}).(\forall V6t2 \in (2^{A_27a}). \\
& \quad (((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a\ 2)\ V0R) \\
& \quad (c_2Emin_2E_3D\ 2))\ V3s1)\ V4s2)) \wedge (p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\
& \quad A_27a\ 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V5t1)\ V6t2))) \Rightarrow ((p\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_pred_set_2EPSUBSETR\ A_27a)\ V0R)\ V3s1)\ V5t1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (ap\ (c_2Equotient_pred_set_2EPSUBSETR\ A_27a)\ V0R) \\
& \quad V4s2)\ V6t2))))))))))))) \\
& \hspace{15em}
\end{aligned}$$