

# thm\_2Equotient\_\_pred\_\_set\_2ESUBSETR\_\_RSP (TMTa9ooegdJWMh1tT2fdj8FsGJ851uk7qve)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 6** We define  $c\_2Equotient\_2EQUOTIENT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1R \in ((2^{A\_27b})^{A\_27b}).$

**Definition 7** We define  $c\_2Equotient\_2E\_3D\_3D\_3D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27a}).\lambda V1R2 \in ((2^{A\_27b})^{A\_27b}).$

**Definition 8** We define  $c\_2Ecombin\_2EW$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x \in A\_27a.(\lambda V2x \in A\_27b.(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (V0f) (V1x)) (V2x))))$

**Definition 9** We define  $c\_2Equotient\_2Erespects$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(c\_2Ecombin\_2EW A\_27a A\_27b)$

**Definition 10** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 11** We define  $c\_2Ebool\_2ERES\_FORALL$  to be  $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).(ap V1m p)))$

**Definition 12** We define  $c\_2Equotient\_pred\_set\_2ESUBSETR$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1R \in ((2^{A\_27a})^{A\_27a}).$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ & (\forall V1x \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V1x) (ap ( \\ & c\_2Equotient\_2Erespects A\_27a 2) V0R))) \Leftrightarrow (p (ap (ap V0R V1x) V1x)))))) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& (\forall V1s \in (2^{A\_27a}). (\forall V2t \in (2^{A\_27a}). ((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E\ A\_27a\ 2)\ V0R)\ (c\_2Emin\_2E\_3D \\
& 2))\ V1s)\ V2t)) \Leftrightarrow (\forall V3x \in A\_27a. (\forall V4y \in A\_27a. ((p\ (ap \\
& (ap\ V0R\ V3x)\ V4y)) \Rightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1s)) \Leftrightarrow \\
& (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V4y)\ V2t)))))))))
\end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1f \in \\
& (2^{A\_27a}). ((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL\ A\_27a)\ V0P)\ V1f)) \Leftrightarrow \\
& (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0P)) \Rightarrow \\
& (p\ (ap\ V1f\ V2x))))))
\end{aligned} \tag{4}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}). ((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3s1 \in (2^{A\_27a}). (\forall V4s2 \in \\
& (2^{A\_27a}). (\forall V5t1 \in (2^{A\_27a}). (\forall V6t2 \in (2^{A\_27a}). \\
& (((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E\ A\_27a\ 2)\ V0R) \\
& (c\_2Emin\_2E\_3D\ 2))\ V3s1)\ V4s2)) \wedge (p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E \\
& A\_27a\ 2)\ V0R)\ (c\_2Emin\_2E\_3D\ 2))\ V5t1)\ V6t2))) \Rightarrow ((p\ (ap\ (ap\ (ap \\
& (c\_2Equotient\_pred\_set\_2ESUBSETR\ A\_27a)\ V0R)\ V3s1)\ V5t1)) \Leftrightarrow \\
& (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_pred\_set\_2ESUBSETR\ A\_27a)\ V0R) \\
& V4s2)\ V6t2)))))))))
\end{aligned}$$