

thm_2Equotient__pred__set_2ESUBSET__PRS
(TMH9QwYcJk3Dy6rQD2XiH7LVQin7TyjcCj6)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)) P))$

Definition 4 We define $c_2Ebool_2E_21$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)) t1 t2))$

Definition 8 We define $c_2Ebool_2E_IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Emin_2E_3D (2^{A_27a})) s t))$

Definition 10 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1f \in (2^{A_27a}).(ap (c_2Emin_2E_3D (2^{A_27a})) f R))$

Definition 11 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 12 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c)^{A_27b})^{A_27a}).(\lambda V1x \in A_27a.(\lambda V2y \in A_27b.V0f x y))$

Definition 13 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a)^{A_27a}) A_27a) A_27a)$

Definition 14 We define $c_2Equotient_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f \in ((A_27c)^{A_27b})^{A_27a}.(\lambda V1x \in A_27a.(\lambda V2y \in A_27b.V0f x y))$

Definition 15 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in ((A_27b)^{A_27a})^{A_27a}).(\lambda V1x \in A_27a.(\lambda V2y \in A_27b.V0f x y))$

Definition 16 We define $c_2Equotient_2Erespects$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(c_2Ecombin_2EW A_27a A_27b)$

Definition 17 We define $c_2Ebool_2ERES_FORALL$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).(ap (c_2Emin_2E_3D (2^{A_27a})) p m)))$

Definition 18 We define $c_2\text{Equotient_pred_set_2ESUBSETR}$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \tag{3}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \tag{4}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1\text{abs} \in (A_27b^{A_27a}).(\forall V2\text{rep} \in (A_27a^{A_27b}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT A_27a A_27b) V0R) V1\text{abs}) V2\text{rep})) \Rightarrow (\forall V3a \in A_27b.((ap V1\text{abs} (ap V2\text{rep} V3a)) = V3a)))))) \tag{5}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1\text{abs} \in (A_27b^{A_27a}).(\forall V2\text{rep} \in (A_27a^{A_27b}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT A_27a A_27b) V0R) V1\text{abs}) V2\text{rep})) \Rightarrow (\forall V3a \in A_27b.(p (ap (ap V0R (ap V2\text{rep} V3a)) (ap V2\text{rep} V3a)))))) \tag{6}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V1x) (ap (c_2Equotient_2Erespects A_27a 2) V0R))) \Leftrightarrow (p (ap (ap V0R V1x) V1x)))) \tag{7}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1s \in (2^{A_27b}).(\forall V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) (ap (ap (ap (c_2Equotient_2E_2D_2D_3E A_27a 2 A_27b 2) V0f) (c_2Ecombin_2EI 2) V1s))) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A_27b) (ap V0f V2x)) V1s)))))) \tag{8}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1f \in \\
& (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ V0P)\ V1f))) \Leftrightarrow \\
& (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0P)) \Rightarrow \\
& (p\ (ap\ V1f\ V2x))))))
\end{aligned} \tag{9}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3s \in (2^{A_27b}). (\forall V4t \in \\
& (2^{A_27b}). ((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27b)\ V3s)\ V4t))) \Leftrightarrow \\
& (p\ (ap\ (ap\ (ap\ (c_2Equotient_pred_set_2ESUBSETR\ A_27a)\ V0R) \\
& (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs) \\
& (c_2Ecombin_2EI\ 2))\ V3s))\ (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI\ 2))\ V4t)))))))))
\end{aligned}$$