

thm\_2Equotient\_\_pred\_\_set\_2EUNIV\_\_PRS  
(TMVp9WdZ6Lgc9isjG1QeUMyZGQMu3VDsps8)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

**Definition 3** We define `c_2Ebool_2ET` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A_27a}))))$

**Definition 5** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V2t \in 2. V2t))))$

**Definition 6** We define `c_2Equotient_2EQUOTIENT` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1f \in (2^{A_27a})$

**Definition 7** We define `c_2Ebool_2EF` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V0t \in 2. V0t))$ .

**Definition 8** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D\_3D\_3E } V0t) (\text{c\_2Ebool\_2EF } 2))))$

**Definition 9** We define `c_2Epred__set_2EUNIV` to be  $\lambda A_27a : \iota. (\lambda V0x \in A_27a. \text{c\_2Ebool\_2EF } 2)$ .

**Definition 10** We define `c_2Ecombin_2EK` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

**Definition 11** We define `c_2Ecombin_2ES` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c)^{A_27b})^{A_27a})$

**Definition 12** We define `c_2Ecombin_2EI` to be  $\lambda A_27a : \iota. (\text{ap } (\text{ap } (\text{c\_2Ecombin\_2ES } A_27a) (A_27a)^{A_27a}) A_27a)$

**Definition 13** We define `c_2Equotient_2E_2D_2D_3E` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda A_27d : \iota. \lambda V0f \in (2^{A_27a})$

**Definition 14** We define `c_2Ebool_2EIN` to be  $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (\text{ap } V1f V0x)))$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (3)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V1t))))))) \quad (4)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(p (ap (ap (c\_2Ebool\_2EIN A\_27a) V0x) (c\_2Epred\_set\_2EUNIV A\_27a)))) \quad (5)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1s \in (2^{A\_27b}).(\forall V2x \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) (ap (ap (ap (c\_2Equotient\_2E\_2D\_2D\_3E A\_27a 2 A\_27b 2) V0f) (c\_2Ecombin\_2EI 2)) V1s))) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN A\_27b) (ap V0f V2x)) V1s)))))) \quad (6)$$

**Theorem 1**

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}).(\forall V2rep \in (A\_27a^{A\_27b}).((p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT A\_27a A\_27b) V0R) V1abs) V2rep)) \Rightarrow ((c\_2Epred\_set\_2EUNIV A\_27b) = (ap (ap (ap (c\_2Equotient\_2E\_2D\_2D\_3E A\_27b 2 A\_27a 2) V2rep) (c\_2Ecombin\_2EI 2)) (c\_2Epred\_set\_2EUNIV A\_27a)))))))$$