

thm_2Equotient__sum_2EINL__PRS (TMVinw4z9LxqcjkKSFeWSULv1u4cRw66Xyi)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 6 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V$

Definition 7 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \tag{1}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{2}$$

Definition 9 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS_sum$

Definition 10 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum$

Let $c_2Esum_2E_2B_2B : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow c_2Esum_2E_2B_2B \\ & A_27a\ A_27b\ A_27c\ A_27d \in (((ty_2Esum_2Esum\ A_27c\ A_27d)^{(ty_2Esum_2Esum\ A_27a\ A_27b)})^{(A_27d^{A_27b})}) \end{aligned} \quad (3)$$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b^{A_27a}). \\ & (\forall V2rep \in (A_27a^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3a \in A_27b. ((ap\ V1abs \\ & (ap\ V2rep\ V3a)) = V3a)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad (\forall V0y \in A_27a. (\forall V1x \in A_27a. (((ap\ (c_2Esum_2EINL \\ & A_27a\ A_27b)\ V1x) = (ap\ (c_2Esum_2EINL\ A_27a\ A_27b)\ V0y)) \Leftrightarrow (V1x = \\ & V0y)))) \wedge (\forall V2y \in A_27b. (\forall V3x \in A_27b. (((ap\ (c_2Esum_2EINR \\ & A_27a\ A_27b)\ V3x) = (ap\ (c_2Esum_2EINR\ A_27a\ A_27b)\ V2y)) \Leftrightarrow (V3x = \\ & V2y)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow ((\forall V0f \in (\\ & \quad A_27c^{A_27a}). (\forall V1g \in (A_27d^{A_27b}). (\forall V2a \in A_27a. \\ & ((ap\ (ap\ (ap\ (c_2Esum_2E_2B_2B\ A_27a\ A_27b\ A_27c\ A_27d)\ V0f)\ V1g) \\ & (ap\ (c_2Esum_2EINL\ A_27a\ A_27b)\ V2a)) = (ap\ (c_2Esum_2EINL\ A_27c \\ & \quad A_27d)\ (ap\ V0f\ V2a)))))) \wedge (\forall V3f \in (A_27c^{A_27a}). (\forall V4g \in \\ & (A_27d^{A_27b}). (\forall V5b \in A_27b. ((ap\ (ap\ (ap\ (c_2Esum_2E_2B_2B \\ & A_27a\ A_27b\ A_27c\ A_27d)\ V3f)\ V4g) (ap\ (c_2Esum_2EINR\ A_27a\ A_27b) \\ & \quad V5b)) = (ap\ (c_2Esum_2EINR\ A_27c\ A_27d)\ (ap\ V4g\ V5b)))))) \end{aligned} \quad (8)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\ & \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\ & \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\ & \quad V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\ & \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (\forall V6a \in A_27c.((ap\ (\\ & \quad c_2Esum_2EINL\ A_27c\ A_27d)\ V6a) = (ap\ (ap\ (ap\ (c_2Esum_2E_2B_2B \\ & \quad A_27a\ A_27b\ A_27c\ A_27d)\ V1abs1)\ V4abs2)\ (ap\ (c_2Esum_2EINL\ A_27a \\ & \quad A_27b)\ (ap\ V2rep1\ V6a)))))))))) \end{aligned}$$