# thm_2Equotient__sum_2EISR__PRS (TMUpHwr1m1eFRLkTr4RugD96ioHednD6JUg)

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**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 3** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

**Definition 4** *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A_27}$

**Definition 5** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

**Definition 6** *We define* c_2Equotient_2EQUOTIENT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

**Definition 7** *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V0t \in 2.V0t))$.

**Definition 8** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

**Definition 9** *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 $A$

**Definition 10** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

**Definition 11** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{1}$$

Let $c\_2Esum\_2EISR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EISR\ A\_27a\ A\_27b \in (2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \tag{2}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{3}$$

**Definition 12** *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

**Definition 13** *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Esum\_2E\_2B\_2B : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow c\_2Esum\_2E\_2B\_2B\ A\_27a\ A\_27b\ A\_27c\ A\_27d \in ((((ty\_2Esum\_2Esum\ A\_27c\ A\_27d)^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)})^{(A\_27d^{A\_27b})} \tag{4}$$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{7}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0ss \in (ty\_2Esum\_2Esum\ A\_27a\ A\_27b).((\exists V1x \in A\_27a.(V0ss = (ap\ (\mathsf{c\_2Esum\_2EINL}\ A\_27a\ A\_27b)\ V1x))) \vee (\exists V2y \in A\_27b.(V0ss = (ap\ (\mathsf{c\_2Esum\_2EINR}\ A\_27a\ A\_27b)\ V2y))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0x \in A\_27b.(p\ (ap\ (c\_2Esum\_2EISR\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Esum\_2EINR}\ A\_27a\ A\_27b)\ V0x)))) \wedge (\forall V1y \in A\_27a.(\neg(p\ (ap\ (c\_2Esum\_2EISR\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Esum\_2EINL}\ A\_27a\ A\_27b)\ V1y)))))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow ((\forall V0f \in (A\_27c^{A\_27a}).(\forall V1g \in (A\_27d^{A\_27b}).(\forall V2a \in A\_27a.\ ((ap\ (ap\ (ap\ (c\_2Esum\_2E\_2B\_2B\ A\_27a\ A\_27b\ A\_27c\ A\_27d)\ V0f)\ V1g)\ (ap\ (\mathsf{c\_2Esum\_2EINL}\ A\_27a\ A\_27b)\ V2a)) = (ap\ (\mathsf{c\_2Esum\_2EINL}\ A\_27c\ A\_27d)\ (ap\ V0f\ V2a))))))\wedge(\forall V3f \in (A\_27c^{A\_27a}).(\forall V4g \in (A\_27d^{A\_27b}).(\forall V5b \in A\_27b.((ap\ (ap\ (ap\ (c\_2Esum\_2E\_2B\_2B\ A\_27a\ A\_27b\ A\_27c\ A\_27d)\ V3f)\ V4g)\ (ap\ (\mathsf{c\_2Esum\_2EINR}\ A\_27a\ A\_27b)\ V5b)) = (ap\ (\mathsf{c\_2Esum\_2EINR}\ A\_27c\ A\_27d)\ (ap\ V4g\ V5b)))))))) \tag{12}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6a \in (ty\_2Esum\_2Esum\ A\_27c\ A\_27d).((p\ (ap\ (c\_2Esum\_2EISR\ A\_27c\ A\_27d)\ V6a)) \Leftrightarrow (p\ (ap\ (c\_2Esum\_2EISR\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (c\_2Esum\_2E\_2B\_2B\ A\_27c\ A\_27d\ A\_27a\ A\_27b)\ V2rep1)\ V5rep2)\ V6a)))))))))))))$$