thm_2EratRing_2ERAT__IS__RING (TMVQDPafDiH79WfAjkKUrqnaUB5LrGbhFMW)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 5 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 6 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 7 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 8 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 9 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{7}$$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{9}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{10}$$

Definition 12 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $ty\_2Efrac\_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{11}$$

Let $c\_2Efrac\_2Eabs\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Eabs\_\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)}) \tag{12}$$

Definition 13 *We define* c_2Efrac_2Efrac__1 *to be* $(ap\ c_2Efrac_2Eabs__frac\ (ap\ (ap\ ($c_2Epair_2E_2C$\ ty_2$

Let $c\_2Efrac\_2Erep\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Erep\_\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{13}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{14}$$

Definition 14 *We define* c_2Efrac_2Efrac__dnm *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ t$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{15}$$

Definition 15 *We define* c_2Efrac_2Efrac__nmr *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{16}$$

Definition 16 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 17 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ t} \tag{17}$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{18}$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \tag{19}$$

Definition 18 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 19 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Eintege$

Definition 20 *We define* c_2Erat_2Erat__equiv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Let $ty\_2Erat\_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erat\_2Erat \tag{20}$$

Let $c\_2Erat\_2Eabs\_\_rat\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Eabs\_\_rat\_\_CLASS \in (ty\_2Erat\_2Erat^{(2^{ty\_2Efrac\_2Efrac})}) \tag{21}$$

Definition 21 *We define* c_2Erat_2Eabs__rat *to be* $\lambda V0r \in ty\_2Efrac\_2Efrac.(ap\ c\_2Erat\_2Eabs\_\_rat\_\_Cl$

Definition 22 *We define* c_2Erat_2Erat__1 *to be* $(ap$ c_2Erat_2Eabs__rat c_2Efrac_2Efrac__1).

Let $c\_2Erat\_2Erep\_\_rat\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Erep\_\_rat\_\_CLASS \in ((2^{ty\_2Efrac\_2Efrac})^{ty\_2Erat\_2Erat}) \tag{22}$$

Definition 23 *We define* c_2Erat_2Erep__rat *to be* $\lambda V0a \in ty\_2Erat\_2Erat.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Efrac\_$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{23}$$

Definition 24 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteg$

Definition 25 *We define* c_2Efrac_2Efrac__add *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2E$

Definition 26 *We define* c_2Erat_2Erat__add *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 27 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 28 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 29 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

Let $c\_2Earithmetic\_2Enum\_\_CASE : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2Enum\_\_CASE\ A\_27a \in (((A\_27a^{(A\_27a^{ty\_2Enum\_2Enum})})^{A\_27a})^{ty\_2Enum\_2Enum}) \tag{24}$$

Definition 30 *We define* c_2Efrac_2Efrac__0 *to be* $(ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2$

Definition 31 *We define* c_2Erat_2Erat__0 *to be* $(ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ \mathsf{c\_2Efrac\_2Efrac\_\_0})$.

Definition 32 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 33 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 34 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}$

Let $c\_2Ebool\_2EARB : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{25}$$

Definition 35 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 36 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Definition 37 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 38 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 39 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 40 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 41 *We define* c_2Erat_2Erat__of__num *to be* $(ap\ (ap$ (c_2Erelation_2EWFREC ty_2Enum_2Enum

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{26}$$

Definition 42 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint

Definition 43 *We define* c_2Efrac_2Efrac__ainv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_\_fr$

Definition 44 *We define* c_2Erat_2Erat__ainv *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.(ap$ c_2Erat_2Eabs__rat $(ap$ c_2

Definition 45 *We define* c_2Efrac_2Efrac__mul *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Definition 46 *We define* c_2Erat_2Erat__mul *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Let $ty\_2Ering\_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ering\_2Ering\ A0) \tag{27}$$

Let $c\_2Ering\_2Erecordtype\_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Erecordtype\_2Ering\ A\_27a \in ((((((ty\_2Ering\_2Ering\ A\_27a)^{(A\_27a^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \tag{28}$$

Let $c\_2Ering\_2Ering\_\_RN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RN\ A\_27a \in ((A\_27a^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{29}$$

Let $c\_2Ering\_2Ering\_\_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R1\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{30}$$

Let $c\_2Ering\_2Ering\_\_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R0\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{31}$$

Let $c\_2Ering\_2Ering\_\_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{32}$$

Let $c\_2Ering\_2Ering\_\_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{33}$$

Definition 47 *We define* c_2Ering_2Eis__ring *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(ap\ (ap\ \mathsf{c}$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{36}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.(\forall V2c \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0a)\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V1b)\ V2c)) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0a)\ V1b))\ V2c))))) \tag{37}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.(\forall V2c \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V0a)\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V1b)\ V2c)) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V0a)\ V1b))\ V2c))))) \tag{38}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0a)\ V1b) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V1b)\ V0a)))) \tag{39}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V0a)\ V1b) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V1b)\ V0a)))) \tag{40}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V0a) = V0a)) \tag{41}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V0a) = V0a)) \tag{42}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0a)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0a)) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))) \tag{43}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.(\forall V2c \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0a)\ V1b))\ V2c) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V0a)\ V2c))\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V1b)\ V2c)))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in A\_27a.(\forall V1a0 \in A\_27a.(\forall V2f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V3f0 \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V4f1 \in (A\_27a^{A\_27a}).((ap\ (c\_2Ering\_2Ering\_\_R0\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V0a)\ V1a0)\ V2f)\ V3f0)\ V4f1)) = V0a)))))) \wedge ((\forall V5a \in A\_27a.(\forall V6a0 \in A\_27a.(\forall V7f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V8f0 \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V9f1 \in (A\_27a^{A\_27a}).((ap\ (c\_2Ering\_2Ering\_\_R1\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V5a)\ V6a0)\ V7f)\ V8f0)\ V9f1)) = V6a0)))))) \wedge ((\forall V10a \in A\_27a.(\forall V11a0 \in A\_27a.(\forall V12f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V13f0 \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V14f1 \in (A\_27a^{A\_27a}).((ap\ (c\_2Ering\_2Ering\_\_RP\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V10a)\ V11a0)\ V12f)\ V13f0)\ V14f1)) = V12f)))))) \wedge ((\forall V15a \in A\_27a.(\forall V16a0 \in A\_27a.(\forall V17f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V18f0 \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V19f1 \in (A\_27a^{A\_27a}).((ap\ (c\_2Ering\_2Ering\_\_RM\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V15a)\ V16a0)\ V17f)\ V18f0)\ V19f1)) = V18f0)))))) \wedge (\forall V20a \in A\_27a.(\forall V21a0 \in A\_27a.(\forall V22f \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V23f0 \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V24f1 \in (A\_27a^{A\_27a}).((ap\ (c\_2Ering\_2Ering\_\_RN\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ A\_27a)\ V20a)\ V21a0)\ V22f)\ V23f0)\ V24f1)) = V24f1)))))))))) \tag{45}$$

Theorem 1

$$(p\ (ap\ (\mathsf{c\_2Ering\_2Eis\_\_ring}\ ty\_2Erat\_2Erat)\ (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ ty\_2Erat\_2Erat)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ \mathsf{c\_2Erat\_2Erat\_\_add})\ \mathsf{c\_2Erat\_2Erat\_\_mul})\ \mathsf{c\_2Erat\_2Erat\_\_ainv})))$$