thm_2Erat_2ERATND__RAT__OF__NUM (TMXHq2D33jKY8UrwoPdDbiqsg942XpH4VVP)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 6 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $ty\_2Efrac\_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{3}$$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{4}$$

Let $c\_2Efrac\_2Eabs\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Eabs\_\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)}) \tag{5}$$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 A

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{6}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{10}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{11}$$

Definition 11 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 14 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 15 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{12}$$

Definition 16 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 17 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 18 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{15}$$

Definition 19 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{16}$$

Definition 20 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 21 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Efrac\_2Erep\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Erep\_\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{17}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{18}$$

Definition 22 *We define* c_2Efrac_2Efrac__dnm *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ t$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{19}$$

Definition 23 *We define* c_2Efrac_2Efrac__nmr *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{20}$$

Definition 24 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ t} \quad (21)$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \quad (22)$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)} \quad (23)$$

Definition 25 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 26 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Eintege$

Definition 27 *We define* c_2Erat_2Erat__equiv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Definition 28 *We define* c_2Equotient_2EQUOTIENT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

Definition 29 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 30 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 31 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (24)$$

Definition 32 *We define* c_2Efrac_2Efrac__1 *to be* $(ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap$ (c_2Epair_2E_2C ty_2

Let $ty\_2Erat\_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erat\_2Erat \quad (25)$$

Let $c\_2Erat\_2Erep\_\_rat\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Erep\_\_rat\_\_CLASS \in ((2^{ty\_2Efrac\_2Efrac})^{ty\_2Erat\_2Erat}) \quad (26)$$

Definition 33 *We define* c_2Erat_2Erep__rat *to be* $\lambda V0a \in ty\_2Erat\_2Erat.(ap$ (c_2Emin_2E_40 $ty\_2Efrac\_$

Definition 34 *We define* c_2Efrac_2Efrac__mul *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2E$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (27)$$

Definition 35 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{28}$$

Definition 36 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Definition 37 *We define* c_2Einteger_2EABS *to be* $\lambda V0n \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap$ (c_2Ebool_2EC

Definition 38 *We define* c_2EintExtension_2ESGN *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap$ (c_2Eboo

Definition 39 *We define* c_2Efrac_2Efrac__sgn *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap$ c_2EintExtension_2ES

Definition 40 *We define* c_2Efrac_2Efrac__minv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_\_f$

Let $c\_2Erat\_2Eabs\_\_rat\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Eabs\_\_rat\_\_CLASS \in (ty\_2Erat\_2Erat^{(2^{ty\_2Efrac\_2Efrac})}) \tag{29}$$

Definition 41 *We define* c_2Erat_2Eabs__rat *to be* $\lambda V0r \in ty\_2Efrac\_2Efrac.(ap\ c\_2Erat\_2Eabs\_\_rat\_\_C$

Definition 42 *We define* c_2Erat_2Erat__minv *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.(ap$ c_2Erat_2Eabs__rat $(ap$ c_

Definition 43 *We define* c_2Efrac_2Efrac__ainv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_\_f$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{30}$$

Definition 44 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Definition 45 *We define* c_2Efrac_2Efrac__add *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2E$

Definition 46 *We define* c_2Efrac_2Efrac__sub *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2E$

Definition 47 *We define* c_2Erat_2Erat__sub *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 48 *We define* c_2Erat_2Erat__sgn *to be* $\lambda V0r \in ty\_2Erat\_2Erat.(ap$ c_2Efrac_2Efrac__sgn $(ap$ c_

Definition 49 *We define* c_2Erat_2Erat__les *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$ (

Definition 50 *We define* c_2Erat_2Erat__leq *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$ (

Definition 51 *We define* c_2Erat_2Erat__add *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 52 *We define* c_2Efrac_2Efrac__0 *to be* $(ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap$ (c_2Epair_2E_2C ty_2

Definition 53 *We define* c_2Erat_2Erat__0 *to be* $(ap$ c_2Erat_2Eabs__rat c_2Efrac_2Efrac__0).

Definition 54 *We define* c_2Erat_2Erat__1 *to be* $(ap$ c_2Erat_2Eabs__rat c_2Efrac_2Efrac__1$)$.

Definition 55 *We define* c_2Einteger_2ENum *to be* $\lambda V0i \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 ty_2E

Definition 56 *We define* c_2Erat_2Erat__ainv *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.(ap$ c_2Erat_2Eabs__rat $(ap$ c_2

Definition 57 *We define* c_2Einteger_2Eint__le *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2E$

Definition 58 *We define* c_2Efrac_2Efrac__div *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Definition 59 *We define* c_2Erat_2Erat__div *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$ c

Let $c\_2Erat\_2ERATN : \iota$ be given. Assume the following.

$$c\_2Erat\_2ERATN \in (ty\_2Einteger\_2Eint^{ty\_2Erat\_2Erat}) \quad (31)$$

Definition 60 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 61 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 62 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap$ (c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Let $c\_2Earithmetic\_2Enum\_\_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2Enum\_\_CASE\ A\_27a \in (((A\_27a^{(A\_27a^{ty\_2Enum\_2Enum})})^{A\_27a})^{ty\_2Enum\_2Enum}) \quad (32)$$

Definition 63 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2E_21

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (33)$$

Definition 64 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Definition 65 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 66 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 67 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 68 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 69 *We define* c_2Erat_2Erat__of__num *to be* $(ap\ (ap$ (c_2Erelation_2EWFREC ty_2Enum_2Enum

Definition 70 *We define* c_2Erat_2Erat__of__int *to be* $\lambda V0i \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap$ (c_2Ebool_2E

Let $c\_2Erat\_2ERATD : \iota$ be given. Assume the following.

$$c\_2Erat\_2ERATD \in (ty\_2Enum\_2Enum^{ty\_2Erat\_2Erat}) \quad (34)$$

Definition 71 *We define* c_2Erat_2Erat__mul *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Assume the following.

$$((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\tag{35}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m)\wedge(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m)\wedge(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))))\tag{36}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m))))\tag{37}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = \mathsf{c\_2Enum\_2E0})\vee(\exists V1n \in ty\_2Enum\_2Enum.(V0m = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))\tag{38}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((\neg(V0n = \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n))))\tag{39}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Enum\_2E0})\ V0n)))\tag{40}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m)))))\tag{41}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m))\tag{42}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0})\wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0})\wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m)\wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m)\wedge(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V1n))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))))))) \tag{43}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p)))))) \tag{44}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V2p))\Leftrightarrow((V0m = \mathsf{c\_2Enum\_2E0})\vee (V1n = V2p)))))) \tag{45}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow((V0m = \mathsf{c\_2Enum\_2E0})\vee(V1n = \mathsf{c\_2Enum\_2E0}))))) \tag{46}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((V0m = V1n)\Leftrightarrow((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m)))))) \tag{47}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))))) \tag{48}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m))))) \quad (49)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(V0m = V1n))\Leftrightarrow((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n))\vee(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m)))))) \quad (50)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0n))) \quad (51)$$

Assume the following.

$$True \quad (52)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \quad (53)$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \quad (55)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \quad (56)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\wedge((p\ V1t2)\wedge(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\wedge(p\ V2t3)))))) \quad (57)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t)\Rightarrow False)\Rightarrow(\neg(p\ V0t)))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t))\Rightarrow((p\ V0t)\Rightarrow False))) \quad (59)$$

Assume the following.

$$(\forall V0t \in 2.(((True{\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\wedge}True){\Leftrightarrow}(p\ V0t)){\wedge}(((False{\wedge}(p\ V0t)){\Leftrightarrow}False){\wedge}((((p\ V0t){\wedge}False){\Leftrightarrow}False){\wedge}(((p\ V0t){\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t)))))) \tag{60}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\vee}(p\ V0t)){\Leftrightarrow}True){\wedge}((((p\ V0t){\vee}True){\Leftrightarrow}True){\wedge}(((False{\vee}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\vee}False){\Leftrightarrow}(p\ V0t)){\wedge}(((p\ V0t){\vee}(p\ V0t)){\Leftrightarrow}(p\ V0t)))))) \tag{61}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Rightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\Rightarrow}True){\Leftrightarrow}True){\wedge}(((False{\Rightarrow}(p\ V0t)){\Leftrightarrow}True){\wedge}((((p\ V0t){\Rightarrow}(p\ V0t)){\Leftrightarrow}True){\wedge}(((p\ V0t){\Rightarrow}False){\Leftrightarrow}(\neg(p\ V0t)))))))) \tag{62}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))){\Leftrightarrow}(p\ V0t))){\wedge}(((\neg True){\Leftrightarrow}False){\wedge}((\neg False){\Leftrightarrow}True))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(V0x = V0x)) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.((V0x = V0x){\Leftrightarrow}True)) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y){\Leftrightarrow}(V1y = V0x)))) \tag{66}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\Leftrightarrow}True){\Leftrightarrow}(p\ V0t)){\wedge}(((False{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(\neg(p\ V0t))){\wedge}(((p\ V0t){\Leftrightarrow}False){\Leftrightarrow}(\neg(p\ V0t))))))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))){\Leftrightarrow}(\exists V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{68}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P){\wedge}(\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))){\Leftrightarrow}(\forall V3x \in A\_27a.((p\ V0P){\wedge}(p\ (ap\ V1Q\ V3x))))))) \tag{69}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \tag{70}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A))))) \tag{71}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{72}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \tag{73}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B))))) \tag{74}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \tag{75}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{76}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{77}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{78}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in A\_27a.(\exists V1x \in A\_27a.(V1x = V0a))) \tag{79}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2)))) \tag{80}$$

Assume the following.

$$(\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1b)) \Rightarrow ((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_nmr}\ (ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V0a)\ V1b))) = V0a)))) \tag{81}$$

Assume the following.

$$(\forall V0a \in ty\_2Efrac\_2Efrac.((ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_mul}\ V0a)\ \mathsf{c\_2Efrac\_2Efrac\_\_1}) = V0a)) \tag{82}$$

Assume the following.

$$((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_minv}\ \mathsf{c\_2Efrac\_2Efrac\_\_1}) = \mathsf{c\_2Efrac\_2Efrac\_\_1}) \tag{83}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))))) \tag{84}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)) \Leftrightarrow (V0m = V1n)))) \tag{85}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))))) \tag{86}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)))\Leftrightarrow((\neg(V0n = \mathsf{c\_2Enum\_2E0}))\vee(\neg(V1m = \mathsf{c\_2Enum\_2E0}))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m))))\Leftrightarrow False)))))) \tag{87}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.((\exists V1n \in ty\_2Enum\_2Enum. ((V0p = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))\wedge(\neg(V1n = \mathsf{c\_2Enum\_2E0}))))\vee ((\exists V2n \in ty\_2Enum\_2Enum.((V0p = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V2n)))\wedge(\neg(V2n = \mathsf{c\_2Enum\_2E0}))))\vee (V0p = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))) \tag{88}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Einteger\_2ENum}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)) = V0n)) \tag{89}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Einteger\_2EABS}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n))) \tag{90}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2EABS}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0p)) = (ap\ \mathsf{c\_2Einteger\_2EABS}\ V0p))) \tag{91}$$

Assume the following.

$$((\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum. (((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))\Leftrightarrow(V0m = V1n))))\wedge((\forall V2x \in ty\_2Einteger\_2Eint.(\forall V3y \in ty\_2Einteger\_2Eint.(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V2x) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V3y))\Leftrightarrow(V2x = V3y))))\wedge(\forall V4n \in ty\_2Enum\_2Enum.(\forall V5m \in ty\_2Enum\_2Enum.((((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V4n) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V5m)))\Leftrightarrow((V4n = \mathsf{c\_2Enum\_2E0})\wedge(V5m = \mathsf{c\_2Enum\_2E0})))\wedge(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V4n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V5m))\Leftrightarrow((V4n = \mathsf{c\_2Enum\_2E0})\wedge(V5m = \mathsf{c\_2Enum\_2E0}))))))))) \tag{92}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = \mathsf{c\_2Enum\_2E0}))) \tag{93}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0}))\wedge(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n))\Rightarrow(p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))))))\Rightarrow(\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))) \tag{94}$$

Assume the following.

$$
\begin{gathered}
((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
\mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in \\
ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap \\
(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum. \\
((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\wedge \\
((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A \\
V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum. \\
(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (\\
ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V7m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A \\
V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
\mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum. \\
((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in \\
ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V11m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP \\
\mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V12n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
(ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in \\
ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \mathsf{c\_2Enum\_2E0}) = \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge \\
((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n)) \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \mathsf{c\_2Enum\_2ESUC} \\
\mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
\mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.(\\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) = \\
\mathsf{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE} \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum. \\
(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge \\
((\forall V20n \in ty\_2Enum\_2Enum.((\mathsf{c\_2Enum\_2E0} = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum. \\
(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V21n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge \\
((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V24n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V25n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E} \\
\mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V29n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\
\mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}
\end{gathered}
$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\Leftrightarrow False)\land ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO})\Leftrightarrow False)\land(((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\Leftrightarrow False)\land((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO})\Leftrightarrow False)\land((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))\Leftrightarrow False)\land((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))\Leftrightarrow False)\land((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))\Leftrightarrow(V0n = V1m))\land(((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))\Leftrightarrow(V0n = V1m)))))))))) \tag{96}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))\Leftrightarrow True)\land(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))\Leftrightarrow True)\land(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\land(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\land(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\land(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow (\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n))))\land((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))))))))) \tag{97}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n))\Leftrightarrow True)\land(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\land(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\land (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\land(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\land(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V0n))))\land((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))))))))) \tag{98}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) \Leftrightarrow (V0m = V1n)))) \tag{99}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ \mathsf{c\_2Enum\_2E0})))) \tag{100}$$

Assume the following.

$$(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ ty\_2Efrac\_2Efrac\ ty\_2Erat\_2Erat)\ \mathsf{c\_2Erat\_2Erat\_\_equiv})\ \mathsf{c\_2Erat\_2Eabs\_\_rat})\ \mathsf{c\_2Erat\_2Erep\_\_rat})) \tag{101}$$

Assume the following.

$$((ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ \mathsf{c\_2Efrac\_2Efrac\_\_1})) \tag{102}$$

Assume the following.

$$((\forall V0x \in ty\_2Efrac\_2Efrac.(\forall V1y \in ty\_2Efrac\_2Efrac. ((ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_mul}\ (ap\ \mathsf{c\_2Erat\_2Erep\_\_rat}\ (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V0x)))\ V1y)) = (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_mul}\ V0x)\ V1y)))))\wedge(\forall V2x \in ty\_2Efrac\_2Efrac. (\forall V3y \in ty\_2Efrac\_2Efrac.((ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_mul}\ V2x)\ (ap\ \mathsf{c\_2Erat\_2Erep\_\_rat}\ (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V3y)))) = (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_mul}\ V2x)\ V3y)))))) \tag{103}$$

Assume the following.

$$(\forall V0f1 \in ty\_2Efrac\_2Efrac.((\neg((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_nmr}\ V0f1)))\Rightarrow((ap\ \mathsf{c\_2Erat\_2Erat\_\_minv}\ (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V0f1)) = (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_minv}\ V0f1))))) \tag{104}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0a)\ V1b) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V1b)\ V0a)))) \tag{105}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. ((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_sub}\ V0r1)\ V1r2) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2))))) \tag{106}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. ((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_div}\ V0r1)\ V1r2) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V0r1)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_minv}\ V1r2))))) \tag{107}$$

Assume the following.

$$((ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) \tag{108}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. ((ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ V1r2)) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2))))) \tag{109}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. (((ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1) = V1r2) \Leftrightarrow (V0r1 = (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2))))) \tag{110}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. (((ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2)) \Leftrightarrow (V0r1 = V1r2)))) \tag{111}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. ((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V0r1)\ V1r2)) \Rightarrow (\neg(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V1r2)\ V0r1)))))) \tag{112}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_leq}\ V0r1)\ V0r1))) \tag{113}$$

Assume the following.

$$(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \tag{114}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. ((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V0r1))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_leq}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V1r2))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ V1r2))))))) \tag{115}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. (\forall V2r3 \in ty\_2Erat\_2Erat.((V0r1 = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_sub}\ V1r2)\ V2r3))\Leftrightarrow((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ V2r3) = V1r2))))) \tag{116}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. (\forall V2r3 \in ty\_2Erat\_2Erat.(((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ V2r3) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V1r2)\ V2r3))\Leftrightarrow(V0r1 = V1r2))))) \tag{117}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. ((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V1r2)\ V0r1))))) \tag{118}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0n \in A\_27a.(((ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Erat\_2Erat\_\_0})\wedge(\forall V1n \in ty\_2Enum\_2Enum. ((ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V1n))\ \mathsf{c\_2Erat\_2Erat\_\_1}))))) \tag{119}$$

Assume the following.

$$(\mathsf{c\_2Erat\_2Erat\_\_0} = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) \tag{120}$$

Assume the following.

$$(\mathsf{c\_2Erat\_2Erat\_\_1} = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \tag{121}$$

Assume the following.

$$(\forall V0i1 \in ty\_2Einteger\_2Eint.(\forall V1i2 \in ty\_2Einteger\_2Eint. (((ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ V0i1) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ V1i2))\Leftrightarrow(V0i1 = V1i2)))) \tag{122}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0x)) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V0x))) \tag{123}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ V0x))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ V1y)) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V1y))))) \tag{124}$$

Assume the following.

$$(\forall V0r \in ty\_2Erat\_2Erat.((V0r = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_div}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ (ap\ c\_2Erat\_2ERATN\ V0r)))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ c\_2Erat\_2ERATD\ V0r))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ c\_2Erat\_2ERATD\ V0r)))\wedge((((ap\ c\_2Erat\_2ERATN\ V0r) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\Rightarrow((ap\ c\_2Erat\_2ERATD\ V0r) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge(\forall V1n\_27 \in ty\_2Einteger\_2Eint.(\forall V2d\_27 \in ty\_2Enum\_2Enum.(((V0r = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_div}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ V1n\_27))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V2d\_27)))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V2d\_27)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ \mathsf{c\_2Einteger\_2EABS}\ (ap\ c\_2Erat\_2ERATN\ V0r)))\ (ap\ \mathsf{c\_2Einteger\_2EABS}\ V1n\_27)))))))))) \tag{125}$$

Assume the following.

$$(\forall V0r \in ty\_2Erat\_2Erat.(\forall V1n\_27 \in ty\_2Einteger\_2Eint.(\forall V2d\_27 \in ty\_2Enum\_2Enum.(((V0r = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_div}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ V1n\_27))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V2d\_27)))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V2d\_27)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ \mathsf{c\_2Einteger\_2EABS}\ (ap\ c\_2Erat\_2ERATN\ V0r)))\ (ap\ \mathsf{c\_2Einteger\_2EABS}\ V1n\_27)))))))) \tag{126}$$

Assume the following.

$$(\forall V0r \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V0r)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ c\_2Erat\_2ERATD\ V0r))) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ (ap\ c\_2Erat\_2ERATN\ V0r)))) \tag{127}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{128}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{129}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{130}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{131}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{132}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{133}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{134}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{135}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{136}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \tag{137}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{138}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{139}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{140}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{141}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{142}$$

Theorem 1

$$(\forall V0n \in ty\_2Enum\_2Enum.(((ap\ c\_2Erat\_2ERATN\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V0n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n))\wedge((ap\ c\_2Erat\_2ERATD\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V0n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))$$