

# thm\_2Erat\_2ERATN\_EQ0 (TMG- WXXwmn9s8F78ShfuBtLjMrLBfdFkKiYq)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow Q)$  of type  $\iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 6** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod$

Let  $ty\_2Efrac\_2Efrac : \iota$  be given. Assume the following.

$$nonempty ty\_2Efrac\_2Efrac \tag{3}$$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty ty\_2Einteger\_2Eint \tag{4}$$

Let  $c\_2Efrac\_2Eabs\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Eabs\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod ty\_2Einteger\_2Eint ty\_2Einteger\_2Eint)}) \tag{5}$$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap P x))$  **then**  $(the (\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$   
Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (6)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 9** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 11** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

**Definition 12** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 13** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 14** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 15** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (12)$$

**Definition 16** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 17** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 18** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2E$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (14)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (15)$$

**Definition 19** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (16)$$

**Definition 20** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 21** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Efrac\_2Erep\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Erep\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \quad (17)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (18)$$

**Definition 22** We define  $c\_2Efrac\_2Efrac\_dnm$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap (c\_2Epair\_2ESND\ t$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (19)$$

**Definition 23** We define  $c\_2Efrac\_2Efrac\_nmr$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap (c\_2Epair\_2EFST\ ty$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint}) \quad (20)$$

**Definition 24** We define  $c\_Einteger\_Eint\_REP$  to be  $\lambda V0a \in ty\_Einteger\_Eint.(ap (c\_Emin\_E40 (t$   
 Let  $c\_Einteger\_Eint\_mul : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_mul \in (((ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)_{(ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)}))_{(ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)} \quad (21)$$

Let  $c\_Einteger\_Eint\_eq : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_eq \in ((2^{(ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)}))_{(ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)} \quad (22)$$

Let  $c\_Einteger\_Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_ABS\_CLASS \in (ty\_Einteger\_Eint)^{2^{(ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)}} \quad (23)$$

**Definition 25** We define  $c\_Einteger\_Eint\_ABS$  to be  $\lambda V0r \in (ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum ty\_Eenum\_Eenum)$

**Definition 26** We define  $c\_Einteger\_Eint\_mul$  to be  $\lambda V0T1 \in ty\_Einteger\_Eint.\lambda V1T2 \in ty\_Einteger\_Eint$

**Definition 27** We define  $c\_Erat\_Erat\_equiv$  to be  $\lambda V0f1 \in ty\_Efrac\_Efrac.\lambda V1f2 \in ty\_Efrac\_Efrac$

**Definition 28** We define  $c\_Equotient\_EQUOTIENT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

**Definition 29** We define  $c\_Earithmetic\_EZERO$  to be  $c\_Eenum\_E0$ .

**Definition 30** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_Eenum\_Eenum.(ap (ap c\_Earithmetic\_EBIT1$

**Definition 31** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_Eenum\_Eenum.V0x$ .

Let  $c\_Einteger\_Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_of\_num \in (ty\_Einteger\_Eint)^{ty\_Eenum\_Eenum} \quad (24)$$

**Definition 32** We define  $c\_Efrac\_Efrac\_1$  to be  $(ap c\_Efrac\_Efrac\_abs\_frac (ap (ap (c\_Epair\_E2C ty\_Eenum\_Eenum$

Let  $ty\_Erat\_Erat : \iota$  be given. Assume the following.

$$nonempty\ ty\_Erat\_Erat \quad (25)$$

Let  $c\_Erat\_Erep\_rat\_CLASS : \iota$  be given. Assume the following.

$$c\_Erat\_Erep\_rat\_CLASS \in ((2^{ty\_Efrac\_Efrac})^{ty\_Erat\_Erat}) \quad (26)$$

**Definition 33** We define  $c\_Erat\_Erep\_rat$  to be  $\lambda V0a \in ty\_Erat\_Erat.(ap (c\_Emin\_E40 ty\_Efrac\_Efrac$

**Definition 34** We define  $c\_Efrac\_Efrac\_mul$  to be  $\lambda V0f1 \in ty\_Efrac\_Efrac.\lambda V1f2 \in ty\_Efrac\_Efrac$

Let  $c\_Einteger\_Eint\_neg : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_neg \in ((ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)_{(ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)}) \quad (27)$$

**Definition 35** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap\ c\_2Einteger\_2Eint$ .

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)) \quad (28)$$

**Definition 36** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$ .

**Definition 37** We define  $c\_2Einteger\_2EABS$  to be  $\lambda V0n \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap\ (c\_2Ebool\_2EC$

**Definition 38** We define  $c\_2EintExtension\_2ESGN$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap\ (c\_2Ebool$

**Definition 39** We define  $c\_2Efrac\_2Efrac\_sgn$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2EintExtension\_2ES$

**Definition 40** We define  $c\_2Efrac\_2Efrac\_minv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_f$

Let  $c\_2Erat\_2Eabs\_rat\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erat\_2Eabs\_rat\_CLASS \in (ty\_2Erat\_2Erat^{(2^{ty\_2Efrac\_2Efrac})}) \quad (29)$$

**Definition 41** We define  $c\_2Erat\_2Eabs\_rat$  to be  $\lambda V0r \in ty\_2Efrac\_2Efrac.(ap\ c\_2Erat\_2Eabs\_rat\_CL$

**Definition 42** We define  $c\_2Erat\_2Erat\_minv$  to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.(ap\ c\_2Erat\_2Eabs\_rat\ (ap\ c$

**Definition 43** We define  $c\_2Efrac\_2Efrac\_ainv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_f$

Let  $c\_2Einteger\_2Etint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (30)$$

**Definition 44** We define  $c\_2Einteger\_2Eint\_add$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Eintege$

**Definition 45** We define  $c\_2Efrac\_2Efrac\_add$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

**Definition 46** We define  $c\_2Efrac\_2Efrac\_sub$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

**Definition 47** We define  $c\_2Erat\_2Erat\_sub$  to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

**Definition 48** We define  $c\_2Erat\_2Erat\_sgn$  to be  $\lambda V0r \in ty\_2Erat\_2Erat.(ap\ c\_2Efrac\_2Efrac\_sgn\ (ap\ c$

**Definition 49** We define  $c\_2Erat\_2Erat\_les$  to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

**Definition 50** We define  $c\_2Erat\_2Erat\_leq$  to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

**Definition 51** We define  $c\_2Erat\_2Erat\_add$  to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

**Definition 52** We define  $c\_2Efrac\_2Efrac\_0$  to be  $(ap\ c\_2Efrac\_2Eabs\_frac\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2$

**Definition 53** We define  $c\_2Erat\_2Erat\_0$  to be  $(ap\ c\_2Erat\_2Eabs\_rat\ c\_2Efrac\_2Efrac\_0)$ .

**Definition 54** We define  $c\_2Erat\_2Erat\_1$  to be  $(ap\ c\_2Erat\_2Eabs\_rat\ c\_2Efrac\_2Efrac\_1)$ .

**Definition 55** We define  $c\_2Einteger\_2ENum$  to be  $\lambda V0i \in ty\_2Einteger\_2Eint.(ap\ (c\_2Emin\_2E\_40\ ty\_2Eint))$

**Definition 56** We define  $c\_2Erat\_2Erat\_ainv$  to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.(ap\ c\_2Erat\_2Eabs\_rat\ (ap\ c\_2Eint))$

**Definition 57** We define  $c\_2Einteger\_2Eint\_le$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2Eint$

**Definition 58** We define  $c\_2Efrac\_2Efrac\_div$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

**Definition 59** We define  $c\_2Erat\_2Erat\_div$  to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap\ c\_2Eint)$

Let  $c\_2Erat\_2ERATN : \iota$  be given. Assume the following.

$$c\_2Erat\_2ERATN \in (ty\_2Einteger\_2Eint^{ty\_2Erat\_2Erat}) \quad (31)$$

**Definition 60** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 61** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 62** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap\ (ap\ (c\_2Ecombin\_2ES\ A\_27a\ (A\_27a^{A\_27a}))\ A\_27a))$

Let  $c\_2Earithmetic\_2Enum\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2Enum\_CASE\ A\_27a \in ((A\_27a^{(A\_27a^{ty\_2Enum\_2Enum})})_{A\_27a})_{ty\_2Enum\_2Enum} \quad (32)$$

**Definition 63** We define  $c\_2ERelation\_2EWF$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (c\_2Ebool\_2E\_2E))$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (33)$$

**Definition 64** We define  $c\_2ERelation\_2ERESTRICT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1M \in (A\_27a^{A\_27b})$

**Definition 65** We define  $c\_2ERelation\_2ETC$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b \in A\_27a$

**Definition 66** We define  $c\_2ERelation\_2Eapprox$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M \in (A\_27a^{A\_27b})$

**Definition 67** We define  $c\_2ERelation\_2Ethe\_fun$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M \in (A\_27a^{A\_27b})$

**Definition 68** We define  $c\_2ERelation\_2EWFREC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M \in (A\_27a^{A\_27b})$

**Definition 69** We define  $c\_2Erat\_2Erat\_of\_num$  to be  $(ap\ (ap\ (c\_2ERelation\_2EWFREC\ ty\_2Enum\_2Enum)))$

**Definition 70** We define  $c\_2Erat\_2Erat\_of\_int$  to be  $\lambda V0i \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap\ (c\_2Ebool\_2EARB))))$

Let  $c\_2Erat\_2ERATD : \iota$  be given. Assume the following.

$$c\_2Erat\_2ERATD \in (ty\_2Enum\_2Enum^{ty\_2Erat\_2Erat}) \quad (34)$$

**Definition 71** We define  $c\_Erat\_2Erat\_mul$  to be  $\lambda V0r1 \in ty\_2Erat\_2Erat. \lambda V1r2 \in ty\_2Erat\_2Erat. (ap$

Assume the following.

$$\begin{aligned} & ((ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)) = \\ & \quad (ap\ c\_2Enum\_2ESUC\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\ & \quad \quad c\_2Earithmetic\_2EZERO)))) \end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \quad ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Enum\_2E0)\ V0m) = V0m) \wedge ((ap\ ( \\ & \quad ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ c\_2Enum\_2E0) = V0m) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ & \quad (ap\ c\_2Enum\_2ESUC\ V0m))\ V1n) = (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ & \quad \quad V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ c\_2Enum\_2ESUC \\ & \quad \quad V1n)) = (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))) \end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \quad (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ & \quad \quad V1n)\ V0m)))) \end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. ((V0m = c\_2Enum\_2E0) \vee (\exists V1n \in \\ & \quad ty\_2Enum\_2Enum. (V0m = (ap\ c\_2Enum\_2ESUC\ V1n)))))) \end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum. ((\neg(V0n = c\_2Enum\_2E0)) \Leftrightarrow (p\ (ap\ ( \\ & \quad ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V0n)))) \end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum. (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\ & \quad \quad c\_2Enum\_2E0)\ V0n))) \end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \quad (\neg(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\ & \quad \quad V1n)\ V0m)))))) \end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m) \\ & \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) = \\ & \quad \quad V0m)) \end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
& (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
& ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2E\_2A V0m) \\
& V1n) = (ap (ap c\_2Earithmetic\_2E\_2A V0m) V2p)) \Leftrightarrow ((V0m = c\_2Enum\_2E0) \vee \\
& (V1n = V2p))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n) = c\_2Enum\_2E0) \Leftrightarrow ((V0m = \\
& c\_2Enum\_2E0) \vee (V1n = c\_2Enum\_2E0))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p ( \\
& ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))))
\end{aligned} \tag{48}$$



Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n)) V0m)))))) \quad (49)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n)) V0m)))))) \quad (50)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0n))) \quad (51)$$

Assume the following.

$$True \quad (52)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (53)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (55)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (56)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (57)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (59)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (60)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (61)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (62)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (63)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (64)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (65)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (66)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (67)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (68)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p \ V0P) \wedge (\forall V2x \in A\_27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x)))))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))))))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))))))) \quad (73)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)))))) \quad (74)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow \text{False}) \Leftrightarrow ((p V0t) \Leftrightarrow \text{False}))) \quad (75)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (76)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \quad (77)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27}))))))))) \quad (78)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((ap (ap (ap (c_{.2}Ebool_{.2}ECOND A_{.27a}) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_{.2}Ebool_{.2}ECOND A_{.27a}) V1Q) V3x_{.27} \\ & V5y_{.27})))))))))) \end{aligned} \quad (79)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in A.27a. (\exists V1x \in A.27a. (V1x = V0a))) \quad (80)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((\forall V0t1 \in A.27a. (\forall V1t2 \in \\ & A.27a. ((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ c.2Ebool.2ET)\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A.27a. (\forall V3t2 \in A.27a. ((ap \\ & (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ c.2Ebool.2EF)\ V2t1)\ V3t2) = V3t2)))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty.2Einteger.2Eint. (\forall V1b \in ty.2Einteger.2Eint. \\ & ((p\ (ap\ (ap\ c.2Einteger.2Eint.lt\ (ap\ c.2Einteger.2Eint.of\_num \\ & c.2Enum.2E0)\ V1b)) \Rightarrow ((ap\ c.2Efrac.2Efrac.nmr\ (ap\ c.2Efrac.2Eabs.frac \\ & (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Einteger.2Eint\ ty.2Einteger.2Eint) \\ & V0a)\ V1b))) = V0a)))) \end{aligned} \quad (82)$$

Assume the following.

$$(\forall V0a \in ty.2Efrac.2Efrac. ((ap\ (ap\ c.2Efrac.2Efrac.mul\ V0a)\ c.2Efrac.2Efrac.1) = V0a)) \quad (83)$$

Assume the following.

$$((ap\ c.2Efrac.2Efrac.minv\ c.2Efrac.2Efrac.1) = c.2Efrac.2Efrac.1) \quad (84)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty.2Enum.2Enum. (\forall V1n \in ty.2Enum.2Enum. ( \\ & (p\ (ap\ (ap\ c.2Einteger.2Eint.le\ (ap\ c.2Einteger.2Eint.of\_num \\ & V0m))\ (ap\ c.2Einteger.2Eint.of\_num\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c.2Earithmetic.2E.3C.3D \\ & V0m)\ V1n)))) \end{aligned} \quad (85)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty.2Enum.2Enum. (\forall V1n \in ty.2Enum.2Enum. ( \\ & ((ap\ c.2Einteger.2Eint.of\_num\ V0m) = (ap\ c.2Einteger.2Eint.of\_num \\ & V1n)) \Leftrightarrow (V0m = V1n)))) \end{aligned} \quad (86)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty.2Enum.2Enum. (\forall V1n \in ty.2Enum.2Enum. ( \\ & (ap\ (ap\ c.2Einteger.2Eint.mul\ (ap\ c.2Einteger.2Eint.of\_num \\ & V0m))\ (ap\ c.2Einteger.2Eint.of\_num\ V1n)) = (ap\ c.2Einteger.2Eint.of\_num \\ & (ap\ (ap\ c.2Earithmetic.2E.2A\ V0m)\ V1n)))) \end{aligned} \quad (87)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_of\_num \\
V0n)) (ap c\_2Einteger\_2Eint\_of\_num V1m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_neg \\
& (ap c\_2Einteger\_2Eint\_of\_num V0n)) (ap c\_2Einteger\_2Eint\_neg \\
& (ap c\_2Einteger\_2Eint\_of\_num V1m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V1m) V0n))) \wedge (((p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_neg \\
& (ap c\_2Einteger\_2Eint\_of\_num V0n)) (ap c\_2Einteger\_2Eint\_of\_num \\
V1m))) \Leftrightarrow ((\neg(V0n = c\_2Enum\_2E0)) \vee (\neg(V1m = c\_2Enum\_2E0)))) \wedge ((p \\
& (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_of\_num \\
V0n)) (ap c\_2Einteger\_2Eint\_neg (ap c\_2Einteger\_2Eint\_of\_num \\
V1m)))) \Leftrightarrow False))))))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty\_2Einteger\_2Eint. ((\exists V1n \in ty\_2Enum\_2Enum. \\
& ((V0p = (ap c\_2Einteger\_2Eint\_of\_num V1n)) \wedge (\neg(V1n = c\_2Enum\_2E0)))) \vee \\
& ((\exists V2n \in ty\_2Enum\_2Enum. ((V0p = (ap c\_2Einteger\_2Eint\_neg \\
& (ap c\_2Einteger\_2Eint\_of\_num V2n)) \wedge (\neg(V2n = c\_2Enum\_2E0)))) \vee \\
& (V0p = (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0))))))
\end{aligned} \tag{89}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Einteger\_2EABs (ap c\_2Einteger\_2Eint\_of\_num V0n)) = V0n)) \tag{90}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Einteger\_2EABs (ap c\_2Einteger\_2Eint\_of\_num V0n)) = (ap c\_2Einteger\_2Eint\_of\_num V0n))) \tag{91}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint. ((ap c\_2Einteger\_2EABs (ap c\_2Einteger\_2Eint\_neg V0p)) = (ap c\_2Einteger\_2EABs V0p))) \tag{92}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (((ap\ c\_2Einteger\_2Eint\_of\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_of\_num \\
& V1n)) \Leftrightarrow (V0m = V1n)))) \wedge ((\forall V2x \in ty\_2Einteger\_2Eint. (\forall V3y \in \\
& ty\_2Einteger\_2Eint. (((ap\ c\_2Einteger\_2Eint\_neg\ V2x) = (ap\ c\_2Einteger\_2Eint\_neg \\
& V3y)) \Leftrightarrow (V2x = V3y)))) \wedge (\forall V4n \in ty\_2Enum\_2Enum. (\forall V5m \in \\
& ty\_2Enum\_2Enum. (((ap\ c\_2Einteger\_2Eint\_of\_num\ V4n) = (ap \\
& c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V5m))) \Leftrightarrow \\
& ((V4n = c\_2Enum\_2E0) \wedge (V5m = c\_2Enum\_2E0))) \wedge (((ap\ c\_2Einteger\_2Eint\_neg \\
& (ap\ c\_2Einteger\_2Eint\_of\_num\ V4n)) = (ap\ c\_2Einteger\_2Eint\_of\_num \\
& V5m)) \Leftrightarrow ((V4n = c\_2Enum\_2E0) \wedge (V5m = c\_2Enum\_2E0))))))))) \\
& \tag{93}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\neg((ap\ c\_2Enum\_2ESUC\ V0n) = c\_2Enum\_2E0))) \\
& \tag{94}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (((p\ (ap\ V0P\ c\_2Enum\_2E0)) \wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c\_2Enum\_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum. (p\ (ap\ V0P\ V2n)))))) \\
& \tag{95}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((c\_2Earithmic\_2EZERO = (ap\ c\_2Earithmic\_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = c\_2Earithmic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((c\_2Earithmic\_2EZERO = (ap\ c\_2Earithmic\_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = c\_2Earithmic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = (ap\ c\_2Earithmic\_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = (ap\ c\_2Earithmic\_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = (ap\ c\_2Earithmic\_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = (ap\ c\_2Earithmic\_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
& \tag{97}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Earithmic\_2EZERO)\ (ap\ c\_2Earithmic\_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Earithmic\_2EZERO) \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& V0n)\ c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& (ap\ c\_2Earithmic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& (ap\ c\_2Earithmic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow \\
& (\neg(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))))))))) \\
& \tag{98}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ c\_2Earithmic\_2EZERO)\ V0n)) \Leftrightarrow \\
& True) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT1 \\
& V0n))\ c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))\ c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ (ap\ c\_2Earithmic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& V0n)\ V1m))))))))) \\
& \tag{99}
\end{aligned}$$



Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((ap\ c\_2Enum\_2ESUC\ V0m) = (ap\ c\_2Enum\_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \quad (100)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ c\_2Enum\_2E0)))) \quad (101)$$

Assume the following.

$$(p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ ty\_2Efrac\_2Efrac\ ty\_2Erat\_2Erat)\ c\_2Erat\_2Erat\_equiv)\ c\_2Erat\_2Eabs\_rat)\ c\_2Erat\_2Erep\_rat)) \quad (102)$$

Assume the following.

$$((ap\ c\_2Erat\_2Erat\_of\_num\ (ap\ c\_2Earithmic\_2ENUMERAL\ (ap\ c\_2Earithmic\_2EBIT1\ c\_2Earithmic\_2EZERO))) = (ap\ c\_2Erat\_2Eabs\_rat\ c\_2Efrac\_2Efrac\_1)) \quad (103)$$

Assume the following.

$$((\forall V0x \in ty\_2Efrac\_2Efrac. (\forall V1y \in ty\_2Efrac\_2Efrac. ((ap\ c\_2Erat\_2Eabs\_rat\ (ap\ (ap\ c\_2Efrac\_2Efrac\_mul\ (ap\ c\_2Erat\_2Erep\_rat\ (ap\ c\_2Erat\_2Eabs\_rat\ V0x))\ V1y)) = (ap\ c\_2Erat\_2Eabs\_rat\ (ap\ (ap\ c\_2Efrac\_2Efrac\_mul\ V0x)\ V1y)))))) \wedge (\forall V2x \in ty\_2Efrac\_2Efrac. (\forall V3y \in ty\_2Efrac\_2Efrac. ((ap\ c\_2Erat\_2Eabs\_rat\ (ap\ (ap\ c\_2Efrac\_2Efrac\_mul\ V2x)\ (ap\ c\_2Erat\_2Erep\_rat\ (ap\ c\_2Erat\_2Eabs\_rat\ V3y)))) = (ap\ c\_2Erat\_2Eabs\_rat\ (ap\ (ap\ c\_2Efrac\_2Efrac\_mul\ V2x)\ V3y)))))) \quad (104)$$

Assume the following.

$$(\forall V0f1 \in ty\_2Efrac\_2Efrac. ((\neg ((ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0) = (ap\ c\_2Efrac\_2Efrac\_nmr\ V0f1))) \Rightarrow ((ap\ c\_2Erat\_2Erat\_minv\ (ap\ c\_2Erat\_2Eabs\_rat\ V0f1)) = (ap\ c\_2Erat\_2Eabs\_rat\ (ap\ c\_2Efrac\_2Efrac\_minv\ V0f1)))))) \quad (105)$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat. (\forall V1b \in ty\_2Erat\_2Erat. (ap\ (ap\ c\_2Erat\_2Erat\_add\ V0a)\ V1b) = (ap\ (ap\ c\_2Erat\_2Erat\_add\ V1b)\ V0a)))) \quad (106)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. ((ap\ (ap\ c\_2Erat\_2Erat\_sub\ V0r1)\ V1r2) = (ap\ (ap\ c\_2Erat\_2Erat\_add\ V0r1)\ (ap\ c\_2Erat\_2Erat\_ainv\ V1r2)))))) \quad (107)$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. \\
& ((ap (ap c\_2Erat\_2Erat\_div V0r1) V1r2) = (ap (ap c\_2Erat\_2Erat\_mul \\
& V0r1) (ap c\_2Erat\_2Erat\_minv V1r2))))))
\end{aligned} \tag{108}$$

Assume the following.

$$\begin{aligned}
& ((ap c\_2Erat\_2Erat\_ainv (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) = \\
& (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0))
\end{aligned} \tag{109}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. \\
& ((ap c\_2Erat\_2Erat\_ainv (ap (ap c\_2Erat\_2Erat\_add V0r1) V1r2)) = \\
& (ap (ap c\_2Erat\_2Erat\_add (ap c\_2Erat\_2Erat\_ainv V0r1)) (ap \\
& c\_2Erat\_2Erat\_ainv V1r2))))))
\end{aligned} \tag{110}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. \\
& (((ap c\_2Erat\_2Erat\_ainv V0r1) = V1r2) \Leftrightarrow (V0r1 = (ap c\_2Erat\_2Erat\_ainv \\
& V1r2))))))
\end{aligned} \tag{111}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. \\
& (((ap c\_2Erat\_2Erat\_ainv V0r1) = (ap c\_2Erat\_2Erat\_ainv V1r2)) \Leftrightarrow \\
& (V0r1 = V1r2))))))
\end{aligned} \tag{112}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. \\
& ((p (ap (ap c\_2Erat\_2Erat\_les V0r1) V1r2)) \Rightarrow (\neg (p (ap (ap c\_2Erat\_2Erat\_les \\
& V1r2) V0r1))))))
\end{aligned} \tag{113}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty\_2Erat\_2Erat. (p (ap (ap c\_2Erat\_2Erat\_leq V0r1) \\
& V0r1)))
\end{aligned} \tag{114}$$

Assume the following.

$$\begin{aligned}
& (p (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) \\
& (ap c\_2Erat\_2Erat\_of\_num (ap c\_2Earithmic\_2ENUMERAL (ap \\
& c\_2Earithmic\_2EBIT1 c\_2Earithmic\_2EZERO))))))
\end{aligned} \tag{115}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. \\
& ((p (ap (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) \\
& V0r1)) \Rightarrow ((p (ap (ap (ap c\_2Erat\_2Erat\_leq (ap c\_2Erat\_2Erat\_of\_num \\
& c\_2Enum\_2E0)) V1r2)) \Rightarrow (p (ap (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap c\_2Erat\_2Erat\_add V0r1) V1r2))))))))))
\end{aligned} \tag{116}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. \\
& (\forall V2r3 \in ty\_2Erat\_2Erat. ((V0r1 = (ap (ap c\_2Erat\_2Erat\_sub \\
& V1r2) V2r3)) \Leftrightarrow ((ap (ap c\_2Erat\_2Erat\_add V0r1) V2r3) = V1r2))))))
\end{aligned} \tag{117}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. \\
& (\forall V2r3 \in ty\_2Erat\_2Erat. (((ap (ap c\_2Erat\_2Erat\_add V0r1) \\
& V2r3) = (ap (ap c\_2Erat\_2Erat\_add V1r2) V2r3)) \Leftrightarrow (V0r1 = V1r2))))))
\end{aligned} \tag{118}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. \\
& ((p (ap (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_ainv V0r1)) \\
& (ap c\_2Erat\_2Erat\_ainv V1r2))) \Leftrightarrow (p (ap (ap (ap c\_2Erat\_2Erat\_les \\
& V1r2) V0r1))))))
\end{aligned} \tag{119}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. \\
& (((V0r1 = (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) \vee (V1r2 = (ap \\
& c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0))) \Leftrightarrow ((ap (ap c\_2Erat\_2Erat\_mul \\
& V0r1) V1r2) = (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0))))))
\end{aligned} \tag{120}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0n \in A\_27a. (((ap c\_2Erat\_2Erat\_of\_num \\
& c\_2Enum\_2E0) = c\_2Erat\_2Erat\_0) \wedge (\forall V1n \in ty\_2Enum\_2Enum. \\
& ((ap c\_2Erat\_2Erat\_of\_num (ap c\_2Enum\_2ESUC V1n)) = (ap (ap c\_2Erat\_2Erat\_add \\
& (ap c\_2Erat\_2Erat\_of\_num V1n) c\_2Erat\_2Erat\_1))))))
\end{aligned} \tag{121}$$

Assume the following.

$$(c\_2Erat\_2Erat\_0 = (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) \tag{122}$$

Assume the following.

$$\begin{aligned}
& (c\_2Erat\_2Erat\_1 = (ap c\_2Erat\_2Erat\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))
\end{aligned} \tag{123}$$

Assume the following.

$$(\forall V0i1 \in ty\_2Einteger\_2Eint. (\forall V1i2 \in ty\_2Einteger\_2Eint. ((ap\ c\_2Erat\_2Erat\_of\_int\ V0i1) = (ap\ c\_2Erat\_2Erat\_of\_int\ V1i2)) \Leftrightarrow (V0i1 = V1i2)))) \quad (124)$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum. ((ap\ c\_2Erat\_2Erat\_of\_int\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V0x)) = (ap\ c\_2Erat\_2Erat\_of\_num\ V0x))) \quad (125)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. ((ap\ (ap\ c\_2Erat\_2Erat\_mul\ (ap\ c\_2Erat\_2Erat\_of\_int\ V0x)) (ap\ c\_2Erat\_2Erat\_of\_int\ V1y)) = (ap\ c\_2Erat\_2Erat\_of\_int\ (ap\ (ap\ c\_2Einteger\_2Eint\_mul\ V0x\ V1y)))))) \quad (126)$$

Assume the following.

$$(\forall V0r \in ty\_2Erat\_2Erat. ((V0r = (ap\ (ap\ c\_2Erat\_2Erat\_div\ (ap\ c\_2Erat\_2Erat\_of\_int\ (ap\ c\_2Erat\_2ERATN\ V0r))\ (ap\ c\_2Erat\_2Erat\_of\_num\ (ap\ c\_2Erat\_2ERATD\ V0r)))) \wedge (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ (ap\ c\_2Erat\_2ERATD\ V0r))) \wedge (((ap\ c\_2Erat\_2ERATN\ V0r) = (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0)) \Rightarrow ((ap\ c\_2Erat\_2ERATD\ V0r) = (ap\ c\_2Earithmic\_2ENUMERAL\ (ap\ c\_2Earithmic\_2EBIT1\ c\_2Earithmic\_2EZERO)))) \wedge (\forall V1n\_27 \in ty\_2Einteger\_2Eint. (\forall V2d\_27 \in ty\_2Enum\_2Enum. ((V0r = (ap\ (ap\ c\_2Erat\_2Erat\_div\ (ap\ c\_2Erat\_2Erat\_of\_int\ V1n\_27)) (ap\ c\_2Erat\_2Erat\_of\_num\ V2d\_27))) \wedge (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V2d\_27))) \Rightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Eint\_le\ (ap\ c\_2Einteger\_2EABS\ (ap\ c\_2Erat\_2ERATN\ V0r))\ (ap\ c\_2Einteger\_2EABS\ V1n\_27)))))))))) \quad (127)$$

Assume the following.

$$(\forall V0r \in ty\_2Erat\_2Erat. (\forall V1n\_27 \in ty\_2Einteger\_2Eint. (\forall V2d\_27 \in ty\_2Enum\_2Enum. (((V0r = (ap\ (ap\ c\_2Erat\_2Erat\_div\ (ap\ c\_2Erat\_2Erat\_of\_int\ V1n\_27))\ (ap\ c\_2Erat\_2Erat\_of\_num\ V2d\_27))) \wedge (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V2d\_27))) \Rightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Eint\_le\ (ap\ c\_2Einteger\_2EABS\ (ap\ c\_2Erat\_2ERATN\ V0r))\ (ap\ c\_2Einteger\_2EABS\ V1n\_27)))))))))) \quad (128)$$

Assume the following.

$$(\forall V0r \in ty\_2Erat\_2Erat. ((ap\ (ap\ c\_2Erat\_2Erat\_mul\ V0r)\ (ap\ c\_2Erat\_2Erat\_of\_num\ (ap\ c\_2Erat\_2ERATD\ V0r))) = (ap\ c\_2Erat\_2Erat\_of\_int\ (ap\ c\_2Erat\_2ERATN\ V0r)))) \quad (129)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (130)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (131)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (132)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (133)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (134)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (135)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))) \quad (136)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (137)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (138)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (139)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (140)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (141)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p))))) \quad (142)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (143)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (144)$$

**Theorem 1**

$$(\forall V0r \in ty\_2Erat\_2Erat. (((ap\ c\_2Erat\_2ERATN\ V0r) = (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0)) \Leftrightarrow (V0r = (ap\ c\_2Erat\_2Erat\_of\_num\ c\_2Enum\_2E0))) \wedge (((ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0) = (ap\ c\_2Erat\_2ERATN\ V0r)) \Leftrightarrow (V0r = (ap\ c\_2Erat\_2Erat\_of\_num\ c\_2Enum\_2E0))))))$$