

thm_2Erat_2ERATN__RATD__EQ__THM
(TMLzzgC2SyJ68qPQNtQ4SyStWYEuYvP9xHi)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{3}$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{ty_2Einteger_2Eint}) \tag{4}$$

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_27$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a})))$

Definition 5 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E_40\ (ty_2Einteger_2Eint_REP_CLASS\ a)))$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \tag{5}$$

Let $c_Einteger_Etint_eq : \iota$ be given. Assume the following.

$$c_Einteger_Etint_eq \in ((2^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)}) (ty_Epair_Eprod\ ty_Eenum_Eenum)) \quad (6)$$

Let $c_Einteger_Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Einteger_Eint_ABS_CLASS \in (ty_Einteger_Eint)^{(2^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)})} \quad (7)$$

Definition 6 We define $c_Einteger_Eint_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)$

Definition 7 We define $c_Einteger_Eint_neg$ to be $\lambda V0T1 \in ty_Einteger_Eint.(ap\ c_Einteger_Eint_ABS)$

Let $c_Eenum_EZERO_REP : \iota$ be given. Assume the following.

$$c_Eenum_EZERO_REP \in \omega \quad (8)$$

Let $c_Eenum_EABS_num : \iota$ be given. Assume the following.

$$c_Eenum_EABS_num \in (ty_Eenum_Eenum)^{\omega} \quad (9)$$

Definition 8 We define c_Eenum_E0 to be $(ap\ c_Eenum_EABS_num\ c_Eenum_EZERO_REP)$.

Let $c_Einteger_Eint_of_num : \iota$ be given. Assume the following.

$$c_Einteger_Eint_of_num \in (ty_Einteger_Eint)^{ty_Eenum_Eenum} \quad (10)$$

Let $c_Einteger_Etint_lt : \iota$ be given. Assume the following.

$$c_Einteger_Etint_lt \in ((2^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)}) (ty_Epair_Eprod\ ty_Eenum_Eenum)) \quad (11)$$

Definition 9 We define $c_Einteger_Eint_lt$ to be $\lambda V0T1 \in ty_Einteger_Eint.\lambda V1T2 \in ty_Einteger_Eint$

Definition 10 We define c_Ebool_EF to be $(ap\ (c_Ebool_E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_Emin_E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 12 We define $c_Ebool_E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 13 We define c_Ebool_ECOND to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.27a)))$

Definition 14 We define $c_Einteger_EABS$ to be $\lambda V0n \in ty_Einteger_Eint.(ap\ (ap\ (ap\ (c_Ebool_ECOND\ 2)\ c_Ebool_E21)\ c_Emin_E3D_3D_3E)\ V0n))$

Definition 15 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_Emin_E3D_3D_3E\ V0t)\ c_Ebool_E21))$

Definition 16 We define $c_Einteger_Eint_le$ to be $\lambda V0x \in ty_Einteger_Eint.\lambda V1y \in ty_Einteger_Eint$

Definition 17 We define $c_Earithmic_EZERO$ to be c_Eenum_E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (13)$$

Definition 18 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 19 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 20 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 21 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 22 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Erat_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \quad (15)$$

Let $c_2Erat_2ERATD : \iota$ be given. Assume the following.

$$c_2Erat_2ERATD \in (ty_2Enum_2Enum^{ty_2Erat_2Erat}) \quad (16)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (17)$$

Definition 23 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \quad (18)$$

Let $c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)}) \quad (19)$$

Definition 24 We define $c_2Efrac_2Efrac_1$ to be $(ap\ c_2Efrac_2Eabs_frac\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (20)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (21)$$

Definition 25 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2ESND\ ty_2Efrac_2Efrac\ f))$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (22)$$

Definition 26 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2EFST\ ty_2Efrac_2Efrac\ f))$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (23)$$

Definition 27 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.c_2Einteger_2Etint_mul\ T1\ T2$

Definition 28 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac.c_2Erat_2Eabs_rat_CLASS\ f1\ f2$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat)^{(2^{ty_2Efrac_2Efrac})} \quad (24)$$

Definition 29 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap\ c_2Erat_2Eabs_rat_CLASS\ r)$

Definition 30 We define $c_2Erat_2Erat_1$ to be $(ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_1)$.

Let $c_2Erat_2Erep_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (25)$$

Definition 31 We define $c_2Erat_2Erep_rat$ to be $\lambda V0a \in ty_2Erat_2Erat.(ap\ (c_2Emin_2E40\ ty_2Efrac_2Efrac\ a))$

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (26)$$

Definition 32 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 33 We define $c_2Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 34 We define $c_2Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 35 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 36 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 37 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a}) A$

Let $c_2Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Earithmetic_2Enum_CASE A_27a \in \left((A_27a^{(A_27a^{ty_2Enum_2Enum})})_{A_27a} \right)_{ty_2Enum_2Enum} \quad (27)$$

Definition 38 We define $c_2Efrac_2Efrac_0$ to be $(ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C ty_2$

Definition 39 We define $c_2Erat_2Erat_0$ to be $(ap c_2Erat_2Eabs_rat c_2Efrac_2Efrac_0)$.

Definition 40 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2EARB A_27a \in A_27a \quad (28)$$

Definition 41 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1M$

Definition 42 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b$

Definition 43 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 44 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 45 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 46 We define $c_2Erat_2Erat_of_num$ to be $(ap (ap (c_2Erelation_2EWFREC ty_2Enum_2Enum$

Let $c_2Erat_2ERATN : \iota$ be given. Assume the following.

$$c_2Erat_2ERATN \in (ty_2Einteger_2Eint^{ty_2Erat_2Erat}) \quad (29)$$

Definition 47 We define $c_2Einteger_2ENum$ to be $\lambda V0i \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 ty_2E$

Definition 48 We define $c_2Efrac_2Efrac_ainv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_2Efrac_2Eabs_frac$

Definition 49 We define $c_2Erat_2Erat_ainv$ to be $\lambda V0r1 \in ty_2Erat_2Erat.(ap c_2Erat_2Eabs_rat (ap c_2$

Definition 50 We define $c_Erat_2Erat_of_int$ to be $\lambda V0i \in ty_2Einteger_2Eint.(ap (ap (ap (c_Ebool_2Ebool_of_int V0i) c_Ebool_2Ebool) c_Eint_2Eint) c_Eint_2Eint))$

Definition 51 We define $c_EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint.(ap (ap (ap (c_Ebool_2Ebool_of_int V0x) c_Ebool_2Ebool) c_Eint_2Eint) c_Eint_2Eint))$

Definition 52 We define $c_Efrac_2Efrac_sgn$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_EintExtension_2ESGN V0f1)$

Definition 53 We define $c_Efrac_2Efrac_minv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_Efrac_2Eabs_2Eabs V0f1)$

Definition 54 We define $c_Efrac_2Efrac_mul$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac.(ap c_Efrac_2Efrac_minv (ap c_Efrac_2Efrac_sgn (ap c_Efrac_2Efrac_minv f1) f2)))$

Definition 55 We define $c_Efrac_2Efrac_div$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac.(ap c_Efrac_2Efrac_mul (ap c_Efrac_2Efrac_minv f1) f2))$

Definition 56 We define $c_Erat_2Erat_div$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap c_Efrac_2Efrac_div (ap c_Erat_2Erat_of_int V0r1) V1r2))$

Assume the following.

$$\begin{aligned}
& (\forall V0r \in ty_2Erat_2Erat.((V0r = (ap (ap c_2Erat_2Erat_div \\
& (ap c_2Erat_2Erat_of_int (ap c_2Erat_2ERATN V0r))) (ap c_2Erat_2Erat_of_num \\
& (ap c_2Erat_2ERATD V0r)))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) \\
& (ap c_2Erat_2ERATD V0r))) \wedge (((ap c_2Erat_2ERATN V0r) = (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) \Rightarrow ((ap c_2Erat_2ERATD V0r) = (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge (\forall V1n_27 \in \\
& ty_2Einteger_2Eint.(\forall V2d_27 \in ty_2Enum_2Enum.(((V0r = \\
& (ap (ap c_2Erat_2Erat_div (ap c_2Erat_2Erat_of_int V1n_27)) \\
& (ap c_2Erat_2Erat_of_num V2d_27))) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V2d_27))) \Rightarrow (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2EABS \\
& (ap c_2Erat_2ERATN V0r))) (ap c_2Einteger_2EABS V1n_27))))))))))
\end{aligned}
\tag{30}$$

Theorem 1

$$\begin{aligned}
& (\forall V0r \in ty_2Erat_2Erat.(V0r = (ap (ap c_2Erat_2Erat_div \\
& (ap c_2Erat_2Erat_of_int (ap c_2Erat_2ERATN V0r))) (ap c_2Erat_2Erat_of_num \\
& (ap c_2Erat_2ERATD V0r))))
\end{aligned}$$