thm_2Erat_2ERATN__RATD__MULT (TMXaywSJvbRCqfjWxzQbDxwR9fMtBs4J432)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{3}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 14 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{7}$$

Definition 15 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 17 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap$ (c_2Ebool_2E

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Definition 18 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{11}$$

Definition 19 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 20 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{12}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{13}$$

Let $ty\_2Efrac\_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{14}$$

Let $c\_2Efrac\_2Erep\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Erep\_\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{15}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{16}$$

Definition 21 *We define* c_2Efrac_2Efrac__dnm *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ t$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{17}$$

Definition 22 *We define* c_2Efrac_2Efrac__nmr *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{18}$$

Definition 23 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{19}$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \quad (20)$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \quad (21)$$

Definition 24 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 25 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Definition 26 *We define* c_2Erat_2Erat__equiv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Definition 27 *We define* c_2Equotient_2EQUOTIENT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

Definition 28 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Definition 29 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 30 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (22)$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (23)$$

Definition 31 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Efrac\_2Eabs\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Eabs\_\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)}) \quad (24)$$

Definition 32 *We define* c_2Efrac_2Efrac__1 *to be* $(ap\ c_2Efrac_2Eabs__frac\ (ap\ (ap\ ($c_2Epair_2E_2C ty_2

Let $ty\_2Erat\_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erat\_2Erat \quad (25)$$

Let $c\_2Erat\_2Erep\_\_rat\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Erep\_\_rat\_\_CLASS \in ((2^{ty\_2Efrac\_2Efrac})^{ty\_2Erat\_2Erat}) \quad (26)$$

Definition 33 *We define* c_2Erat_2Erep__rat *to be* $\lambda V0a \in ty_2Erat_2Erat.(ap\ ($c_2Emin_2E_40 $ty\_2Efrac\_$

Definition 34 *We define* c_2Efrac_2Efrac__mul *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2E$

Let $c\_2Erat\_2Eabs\_\_rat\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Eabs\_\_rat\_\_CLASS \in (ty\_2Erat\_2Erat^{(2^{ty\_2Efrac\_2Efrac})}) \tag{27}$$

Definition 35 *We define* c_2Erat_2Eabs__rat *to be* $\lambda V0r \in ty\_2Efrac\_2Efrac.(ap\ c\_2Erat\_2Eabs\_\_rat\_\_C$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{28}$$

Definition 36 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{29}$$

Definition 37 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Definition 38 *We define* c_2Einteger_2EABS *to be* $\lambda V0n \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap$ (c_2Ebool_2EC

Definition 39 *We define* c_2EintExtension_2ESGN *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap$ (c_2Eboo

Definition 40 *We define* c_2Efrac_2Efrac__sgn *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap$ c_2EintExtension_2ES

Definition 41 *We define* c_2Efrac_2Efrac__minv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_\_f$

Definition 42 *We define* c_2Erat_2Erat__minv *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.(ap$ c_2Erat_2Eabs__rat $(ap$ c_

Definition 43 *We define* c_2Erat_2Erat__mul *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 44 *We define* c_2Efrac_2Efrac__ainv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_\_f$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{30}$$

Definition 45 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Integ$

Definition 46 *We define* c_2Efrac_2Efrac__add *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2E$

Definition 47 *We define* c_2Efrac_2Efrac__sub *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2E$

Definition 48 *We define* c_2Erat_2Erat__sub *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 49 *We define* c_2Erat_2Erat__sgn *to be* $\lambda V0r \in ty\_2Erat\_2Erat.(ap$ c_2Efrac_2Efrac__sgn $(ap$ c_

Definition 50 *We define* c_2Erat_2Erat__les *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$ (

Definition 51 *We define* c_2Erat_2Erat__leq *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$ (

Definition 52 *We define* c_2Erat_2Erat__ainv *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.(ap$ c_2Erat_2Eabs__rat $(ap$ c_2

Definition 53 *We define* c_2Erat_2Erat__add *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 54 *We define* c_2Efrac_2Efrac__0 *to be* $(ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap$ (c_2Epair_2E_2C ty_2

Definition 55 *We define* c_2Erat_2Erat__0 *to be* $(ap$ c_2Erat_2Eabs__rat c_2Efrac_2Efrac__0).

Definition 56 *We define* c_2Erat_2Erat__1 *to be* $(ap$ c_2Erat_2Eabs__rat c_2Efrac_2Efrac__1).

Definition 57 *We define* c_2Einteger_2Eint__le *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2E$

Let $c\_2Erat\_2ERATD : \iota$ be given. Assume the following.

$$c\_2Erat\_2ERATD \in (ty\_2Enum\_2Enum^{ty\_2Erat\_2Erat}) \tag{31}$$

Definition 58 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 59 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 60 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap$ (c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Let $c\_2Earithmetic\_2Enum\_\_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2Enum\_\_CASE\ A\_27a \in (((A\_27a^{(A\_27a^{ty\_2Enum\_2Enum})})^{A\_27a})^{ty\_2Enum\_2Enum}) \tag{32}$$

Definition 61 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2E_21

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{33}$$

Definition 62 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1I$

Definition 63 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 64 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 65 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 66 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 67 *We define* c_2Erat_2Erat__of__num *to be* $(ap\ (ap$ (c_2Erelation_2EWFREC ty_2Enum_2Enum

Let $c\_2Erat\_2ERATN : \iota$ be given. Assume the following.

$$c\_2Erat\_2ERATN \in (ty\_2Einteger\_2Eint^{ty\_2Erat\_2Erat}) \tag{34}$$

Definition 68 *We define* c_2Einteger_2ENum *to be* $\lambda V0i \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 ty_2E

Definition 69 *We define* c_2Erat_2Erat__of__int *to be* $\lambda V0i \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap$ (c_2Ebool_2E

Definition 70 *We define* c_2Efrac_2Efrac__div *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Definition 71 *We define* c_2Erat_2Erat__div *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Assume the following.

$$((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \tag{35}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))) \tag{36}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((\neg(V0n = \mathsf{c\_2Enum\_2E0})) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n)))) \tag{37}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Enum\_2E0})\ V0n))) \tag{38}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V1n)) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))))))) \tag{39}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p)))))) \tag{40}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((V0m = V1n)\Leftrightarrow((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m)))))) \tag{41}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))))) \tag{42}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0n))) \tag{43}$$

Assume the following.

$$True \tag{44}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{46}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t)\Rightarrow False)\Rightarrow(\neg(p\ V0t)))) \tag{49}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t))\Rightarrow((p\ V0t)\Rightarrow False))) \tag{50}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow (p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge (((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{51}$$

Assume the following.

$$(\forall V0t \in 2.(((True\vee(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\vee True)\Leftrightarrow True)\wedge (((False\vee(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\vee False)\Leftrightarrow(p\ V0t))\wedge(((p\ V0t)\vee (p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{52}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{53}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge ((\neg False)\Leftrightarrow True))) \tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{56}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{57}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x))))\Leftrightarrow(\exists V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{58}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee((p\ V1B)\vee(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\vee(p\ V2C)))))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A))))) \tag{60}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{61}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \tag{62}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{63}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{64}$$

Assume the following.

$$(\forall V0a \in ty\_2Efrac\_2Efrac.((ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_mul}\ V0a)\ \mathsf{c\_2Efrac\_2Efrac\_\_1}) = V0a)) \tag{65}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = \mathsf{c\_2Enum\_2E0}))) \tag{66}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))) \tag{67}$$

Assume the following.

$$
\begin{gathered}
((\forall V0n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
\mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap} \\
(\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in \\
\mathit{ty\_2Enum\_2Enum}.(\forall V3m \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V3m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\mathit{ap} \\
(\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V2n)\ V3m))))))\wedge((\forall V4n \in \mathit{ty\_2Enum\_2Enum}. \\
((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\wedge \\
((\forall V5n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A} \\
V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in \mathit{ty\_2Enum\_2Enum}. \\
(\forall V7m \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}\ (\\
\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V7m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A} \\
V6n)\ V7m)))))\wedge((\forall V8n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D} \\
\mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in \mathit{ty\_2Enum\_2Enum}. \\
((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in \\
\mathit{ty\_2Enum\_2Enum}.(\forall V11m \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V11m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D} \\
V10n)\ V11m)))))\wedge((\forall V12n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP} \\
\mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V12n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap} \\
(\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ \mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in \\
\mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ V14n)\ \mathsf{c\_2Enum\_2E0}) = \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge \\
((\forall V15n \in \mathit{ty\_2Enum\_2Enum}.(\forall V16m \in \mathit{ty\_2Enum\_2Enum}. \\
((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n)) \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ V15n)\ V16m)))))\wedge(((\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC} \\
\mathsf{c\_2Enum\_2E0}) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
\mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in \mathit{ty\_2Enum\_2Enum}.(\\
(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) = \\
\mathsf{c\_2Enum\_2E0})\wedge((\forall V18n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2EPRE} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in \mathit{ty\_2Enum\_2Enum}. \\
(((\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge \\
((\forall V20n \in \mathit{ty\_2Enum\_2Enum}.((\mathsf{c\_2Enum\_2E0} = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in \mathit{ty\_2Enum\_2Enum}. \\
(\forall V22m \in \mathit{ty\_2Enum\_2Enum}.(((\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V21n) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge \\
((\forall V23n \in \mathit{ty\_2Enum\_2Enum}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow \mathit{False}))\wedge((\forall V24n \in \mathit{ty\_2Enum\_2Enum}. \\
((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V24n)))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V24n))))\wedge((\forall V25n \in \mathit{ty\_2Enum\_2Enum}.(\forall V26m \in \mathit{ty\_2Enum\_2Enum}. \\
((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V25n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V25n)\ V26m)))))\wedge((\forall V27n \in \mathit{ty\_2Enum\_2Enum}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3E} \\
\mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow \mathit{False}))\wedge((\forall V28n \in \mathit{ty\_2Enum\_2Enum}. \\
((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V28n))))\wedge((\forall V29n \in \mathit{ty\_2Enum\_2Enum}.(\forall V30m \in \mathit{ty\_2Enum\_2Enum}. \\
((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V29n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V30m)\ V29n)))))\wedge((\forall V31n \in \mathit{ty\_2Enum\_2Enum}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\
\mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow \mathit{True}))\wedge((\forall V32n \in \mathit{ty\_2Enum\_2Enum}. \\
((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}
\end{gathered}
$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))))))))) \tag{69}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\Leftrightarrow(V0m = V1n)))) \tag{70}$$

Assume the following.

$$(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ ty\_2Efrac\_2Efrac\ ty\_2Erat\_2Erat)\ \mathsf{c\_2Erat\_2Erat\_\_equiv})\ \mathsf{c\_2Erat\_2Eabs\_\_rat})\ \mathsf{c\_2Erat\_2Erep\_\_rat})) \tag{71}$$

Assume the following.

$$((ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ \mathsf{c\_2Efrac\_2Efrac\_\_1})) \tag{72}$$

Assume the following.

$$((\forall V0x \in ty\_2Efrac\_2Efrac.(\forall V1y \in ty\_2Efrac\_2Efrac.((ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_mul}\ (ap\ \mathsf{c\_2Erat\_2Erep\_\_rat}\ (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V0x)))\ V1y)) = (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_mul}\ V0x)\ V1y)))))\wedge(\forall V2x \in ty\_2Efrac\_2Efrac.(\forall V3y \in ty\_2Efrac\_2Efrac.((ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_mul}\ V2x)\ (ap\ \mathsf{c\_2Erat\_2Erep\_\_rat}\ (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V3y)))) = (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_mul}\ V2x)\ V3y)))))) \tag{73}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.(\forall V2c \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V0a) (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V1b)\ V2c)) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul} (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V0a)\ V1b))\ V2c))))) \tag{74}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0a)\ V1b) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add} V1b)\ V0a)))) \tag{75}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.((\neg(V0a = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num} \mathsf{c\_2Enum\_2E0})))\Rightarrow((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_minv} V0a))\ V0a) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \tag{76}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. ((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_sub}\ V0r1)\ V1r2) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add} V0r1)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2))))) \tag{77}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. ((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_div}\ V0r1)\ V1r2) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul} V0r1)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_minv}\ V1r2))))) \tag{78}$$

Assume the following.

$$((ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) \tag{79}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. ((ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ V1r2)) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1))\ (ap \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2))))) \tag{80}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. (((ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1) = V1r2)\Leftrightarrow(V0r1 = (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv} V1r2))))) \tag{81}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.(((ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2)) \Leftrightarrow (V0r1 = V1r2)))) \tag{82}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V0r1)\ V1r2)) \Rightarrow (\neg(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V1r2)\ V0r1)))))) \tag{83}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_leq}\ V0r1)\ V0r1))) \tag{84}$$

Assume the following.

$$(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \tag{85}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V0r1)) \Rightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_leq}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V1r2)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ V1r2)))))))) \tag{86}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.(\forall V2r3 \in ty\_2Erat\_2Erat.((V0r1 = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_sub}\ V1r2)\ V2r3)) \Leftrightarrow ((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ V2r3) = V1r2))))) \tag{87}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.(\forall V2r3 \in ty\_2Erat\_2Erat.(((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ V2r3) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V1r2)\ V2r3)) \Leftrightarrow (V0r1 = V1r2))))) \tag{88}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V1r2)\ V0r1))))) \tag{89}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in A\_27a.(((ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Erat\_2Erat\_\_0}) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V1n))\ \mathsf{c\_2Erat\_2Erat\_\_1})))))) \quad (90)$$

Assume the following.

$$(\mathsf{c\_2Erat\_2Erat\_\_0} = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) \quad (91)$$

Assume the following.

$$(\mathsf{c\_2Erat\_2Erat\_\_1} = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \quad (92)$$

Assume the following.

$$(\forall V0r \in ty\_2Erat\_2Erat.((V0r = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_div}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ (ap\ c\_2Erat\_2ERATN\ V0r)))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ c\_2Erat\_2ERATD\ V0r)))) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ c\_2Erat\_2ERATD\ V0r))) \wedge ((((ap\ c\_2Erat\_2ERATN\ V0r) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \Rightarrow ((ap\ c\_2Erat\_2ERATD\ V0r) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \wedge (\forall V1n\_27 \in ty\_2Einteger\_2Eint.(\forall V2d\_27 \in ty\_2Enum\_2Enum.(((V0r = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_div}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ V1n\_27))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V2d\_27))) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V2d\_27))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ \mathsf{c\_2Einteger\_2EABS}\ (ap\ c\_2Erat\_2ERATN\ V0r)))\ (ap\ \mathsf{c\_2Einteger\_2EABS}\ V1n\_27)))))))))) \quad (93)$$

Assume the following.

$$(\forall V0r \in ty\_2Erat\_2Erat.(V0r = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_div}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ (ap\ c\_2Erat\_2ERATN\ V0r)))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ c\_2Erat\_2ERATD\ V0r))))) \quad (94)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (95)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (96)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \quad (97)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{98}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{99}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))) \tag{100}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{101}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge ((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{102}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p))))))))))) \tag{103}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{104}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{105}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{106}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{107}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{108}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p))) \Rightarrow (p\ V0p))) \tag{109}$$

Theorem 1

$$(\forall V0r \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V0r)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ c\_2Erat\_2ERATD\ V0r))) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ (ap\ c\_2Erat\_2ERATN\ V0r))))$$