

thm_2Erat_2ERATN_SIGN (TM- NxtMY28Vy4NbTy9ngBENuHSVrcRgGxQCA)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2EBOUNDED$ to be $(\lambda V0v \in 2.c_2Ebool_2E_2ET)$.

Definition 4 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p (ap P x))$) of type $\iota \Rightarrow \iota$.

Definition 5 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A P))))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \tag{3}$$

Definition 6 We define $c_2Ebool_2E_2E21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (ap P V0t)) (ap P V0t))))$

Definition 7 We define $c_2Ebool_2E_2E2F$ to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2E2F))$

Definition 10 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 12 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 15 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 18 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 19 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 20 We define $c_2Earithmic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic$

Definition 21 We define $c_2Earithmic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (12)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (13)$$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \quad (14)$$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (15)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (16)$$

Definition 22 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND\ t$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (17)$$

Definition 23 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2EFST\ ty$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (18)$$

Definition 24 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40\ t$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (19)$$

Let $c_Einteger_Etint_eq : \iota$ be given. Assume the following.

$$c_Einteger_Etint_eq \in ((2^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)})^{(ty_Epair_Eprod\ ty_Eenum_Eenum)}) \quad (20)$$

Let $c_Einteger_Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Einteger_Eint_ABS_CLASS \in (ty_Einteger_Eint)^{(2^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)})} \quad (21)$$

Definition 25 We define $c_Einteger_Eint_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)$

Definition 26 We define $c_Einteger_Eint_mul$ to be $\lambda V0T1 \in ty_Einteger_Eint.\lambda V1T2 \in ty_Einteger_Eint$

Definition 27 We define $c_Erat_Erat_equiv$ to be $\lambda V0f1 \in ty_Efrac_Efrac.\lambda V1f2 \in ty_Efrac_Efrac$

Definition 28 We define $c_Equotient_EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda$

Definition 29 We define $c_Earithmetic_EZERO$ to be c_Eenum_E0 .

Definition 30 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_Eenum_Eenum.(ap\ (ap\ c_Earithmetic_EBIT1))$

Definition 31 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_Eenum_Eenum.V0x$.

Let $c_Einteger_Eint_of_num : \iota$ be given. Assume the following.

$$c_Einteger_Eint_of_num \in (ty_Einteger_Eint)^{ty_Eenum_Eenum} \quad (22)$$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_EABS_prod\ A_27a\ A_27b \in ((ty_Epair_Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (23)$$

Definition 32 We define $c_Epair_E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_Epair_E_2C\ ty_27a\ ty_27b))$

Let $c_Efrac_Eabs_frac : \iota$ be given. Assume the following.

$$c_Efrac_Eabs_frac \in (ty_Efrac_Efrac)^{(ty_Epair_Eprod\ ty_Einteger_Eint\ ty_Einteger_Eint)} \quad (24)$$

Definition 33 We define $c_Efrac_Efrac_0$ to be $(ap\ c_Efrac_Eabs_frac\ (ap\ (ap\ (c_Epair_E_2C\ ty_27a\ ty_27b))))$

Definition 34 We define $c_Efrac_Efrac_1$ to be $(ap\ c_Efrac_Eabs_frac\ (ap\ (ap\ (c_Epair_E_2C\ ty_27a\ ty_27b))))$

Let $c_Einteger_Etint_neg : \iota$ be given. Assume the following.

$$c_Einteger_Etint_neg \in ((ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)}) \quad (25)$$

Definition 35 We define $c_Einteger_Eint_neg$ to be $\lambda V0T1 \in ty_Einteger_Eint.(ap\ c_Einteger_Eint_neg)$

Definition 36 We define $c_2Efrac_2Efrac_ainv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap\ c_2Efrac_2Eabs_fn$

Let $ty_2Erat_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \quad (26)$$

Let $c_2Erat_2Erep_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (27)$$

Definition 37 We define $c_2Erat_2Erep_rat$ to be $\lambda V0a \in ty_2Erat_2Erat.(ap\ (c_2Emin_2E40\ ty_2Efrac$

Definition 38 We define $c_2Efrac_2Efrac_mul$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (28)$$

Definition 39 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 40 We define $c_2Einteger_2EABS$ to be $\lambda V0n \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ (c_2Ebool_2EC$

Definition 41 We define $c_2EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ (c_2Eboo$

Definition 42 We define $c_2Efrac_2Efrac_sgn$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap\ c_2EintExtension_2ES$

Definition 43 We define $c_2Efrac_2Efrac_minv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap\ c_2Efrac_2Eabs_fn$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat^{(2^{ty_2Efrac_2Efrac})}) \quad (29)$$

Definition 44 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap\ c_2Erat_2Eabs_rat_CLASS$

Definition 45 We define $c_2Erat_2Erat_minv$ to be $\lambda V0r1 \in ty_2Erat_2Erat.(ap\ c_2Erat_2Eabs_rat\ (ap\ c$

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (30)$$

Definition 46 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 47 We define $c_2Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 48 We define $c_2Efrac_2Efrac_sub$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 49 We define $c_2Erat_2Erat_sub$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 50 We define $c_2Erat_2Erat_sgn$ to be $\lambda V0r \in ty_2Erat_2Erat.(ap\ c_2Efrac_2Efrac_sgn\ (ap\ c_2Erat_2Erat_sgn\ r))$

Definition 51 We define $c_2Erat_2Erat_les$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap\ (c_2Erat_2Erat_les\ r1\ r2))$

Definition 52 We define $c_2Erat_2Erat_leq$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap\ (c_2Erat_2Erat_leq\ r1\ r2))$

Definition 53 We define $c_2Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap\ (c_2Erat_2Erat_add\ r1\ r2))$

Definition 54 We define $c_2Erat_2Erat_0$ to be $(ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_0)$.

Definition 55 We define $c_2Erat_2Erat_1$ to be $(ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_1)$.

Definition 56 We define $c_2Einteger_2ENum$ to be $\lambda V0i \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E40\ ty_2Eint\ i))$

Definition 57 We define $c_2Erat_2Erat_ainv$ to be $\lambda V0r1 \in ty_2Erat_2Erat.(ap\ c_2Erat_2Eabs_rat\ (ap\ c_2Erat_2Erat_ainv\ r1))$

Definition 58 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint.(ap\ (c_2Einteger_2Eint_le\ x\ y))$

Definition 59 We define $c_2Efrac_2Efrac_div$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac.(ap\ (c_2Efrac_2Efrac_div\ f1\ f2))$

Definition 60 We define $c_2Erat_2Erat_div$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap\ (c_2Erat_2Erat_div\ r1\ r2))$

Let $c_2Erat_2ERATN : \iota$ be given. Assume the following.

$$c_2Erat_2ERATN \in (ty_2Einteger_2Eint^{ty_2Erat_2Erat}) \quad (31)$$

Definition 61 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 62 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 63 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a}))\ A_27a))$

Let $c_2Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Earithmetic_2Enum_CASE\ A_27a \in ((A_27a^{(A_27a^{ty_2Enum_2Enum})})^{A_27a})^{ty_2Enum_2Enum} \quad (32)$$

Definition 64 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ (c_2Ebool_2E21\ R))$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (33)$$

Definition 65 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1R \in (A_27a^{A_27b}).(ap\ (c_2Erelation_2ERESTRICT\ f\ R))$

Definition 66 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27a.(ap\ (c_2Erelation_2ETC\ R\ a\ b))$

Definition 67 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).(ap\ (c_2Erelation_2Eapprox\ R\ M))$

Definition 68 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).(ap\ (c_2Erelation_2Ethe_fun\ R\ M))$

Definition 69 We define $c_ERelation_2EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 70 We define $c_ERat_2Erat_of_num$ to be $(ap (ap (c_ERelation_2EWFREC ty_2Enum_2Enum$

Definition 71 We define $c_ERat_2Erat_of_int$ to be $\lambda V0i \in ty_2Einteger_2Eint.(ap (ap (ap (c_Ebool_2E$

Let $c_ERat_2ERATD : \iota$ be given. Assume the following.

$$c_ERat_2ERATD \in (ty_2Enum_2Enum^{ty_2Erat_2Erat}) \quad (34)$$

Definition 72 We define $c_ERat_2Erat_mul$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Assume the following.

$$\begin{aligned} & ((ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)) = \\ & (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n)))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.((V0m = c_2Enum_2E0) \vee (\exists V1n \in \\ & ty_2Enum_2Enum.(V0m = (ap c_2Enum_2ESUC V1n)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.((\neg(V0n = c_2Enum_2E0)) \Leftrightarrow (p (ap (\\ & ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.((\neg(p (ap (ap c_2Eprim_rec_2E_3C \\ & c_2Enum_2E0) V0n))) \Leftrightarrow (V0n = c_2Enum_2E0))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D \\ & c_2Enum_2E0) V0n))) \end{aligned} \quad (41)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (42)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0m)) \quad (43)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))))))) \quad (44)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.(((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \quad (45)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A V0m) V2p)) \Leftrightarrow ((V0m = c_2Enum_2E0) \vee (V1n = V2p)))))) \quad (46)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2A V0m) V1n) = c_2Enum_2E0) \Leftrightarrow ((V0m = c_2Enum_2E0) \vee (V1n = c_2Enum_2E0)))))) \quad (47)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (48)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \quad (49)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m)))) \quad (50)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m)))))) \quad (51)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0n))) \quad (52)$$

Assume the following.

$$True \quad (53)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (55)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (56)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (57)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge ((p \ V1t2) \wedge (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (59)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (60)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (61)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (62)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (63)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (64)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (65)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (66)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (67)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (68)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\forall V1x \in A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (69)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p \ V0P) \wedge (\forall V2x \in A.27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x)))))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\neg((p \ V0A) \Rightarrow (p \ V1B))) \Leftrightarrow ((p \ V0A) \wedge \neg(p \ V1B)))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee (p \ V0A)))))) \quad (73)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \vee \neg(p \ V1B))) \wedge (\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge \neg(p \ V1B)))))) \quad (74)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (p \ V1B)) \wedge (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C)))))) \quad (75)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \Rightarrow (p \ V1B)) \Leftrightarrow ((\neg(p \ V0A)) \vee (p \ V1B)))))) \quad (76)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Leftrightarrow ((p \ V0t) \Leftrightarrow False))) \quad (77)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (78)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \quad (79)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (80)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}. (\forall V3x_{.27} \in A_{.27a}. (\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((ap (ap (ap (c_{.2E}bool_{.2E}COND A_{.27a} \\ & V0P) V2x) V4y) = (ap (ap (ap (c_{.2E}bool_{.2E}COND A_{.27a} V1Q) V3x_{.27} \\ & V5y_{.27})))))))))) \quad (81) \end{aligned}$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0a \in A_{.27a}. (\exists V1x \in A_{.27a}. (V1x = V0a))) \quad (82)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow ((\forall V0t1 \in A_{.27a}. (\forall V1t2 \in \\ & A_{.27a}. ((ap (ap (ap (c_{.2E}bool_{.2E}COND A_{.27a}) c_{.2E}bool_{.2E}ET) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}. (\forall V3t2 \in A_{.27a}. ((ap \\ & (ap (ap (c_{.2E}bool_{.2E}COND A_{.27a}) c_{.2E}bool_{.2E}EF) V2t1) V3t2) = V3t2)))) \quad (83) \end{aligned}$$

Assume the following.

$$(\forall V0v \in 2. ((p (ap c_{.2E}bool_{.2E}BOUNDED V0v)) \Leftrightarrow True)) \quad (84)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty_{.2E}integer_{.2E}int. (\forall V1b \in ty_{.2E}integer_{.2E}int. \\ & ((p (ap (ap c_{.2E}integer_{.2E}int_{.2E}lt (ap c_{.2E}integer_{.2E}int_{.2E}of_{.2E}num \\ & c_{.2E}enum_{.2E}E0)) V1b)) \Rightarrow ((ap c_{.2E}frac_{.2E}frac_{.2E}nmr (ap c_{.2E}frac_{.2E}frac_{.2E}abs_{.2E}frac \\ & (ap (ap (c_{.2E}pair_{.2E}2C ty_{.2E}integer_{.2E}int ty_{.2E}integer_{.2E}int) \\ & V0a) V1b))) = V0a)))) \quad (85) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Einteger_2Eint. (\forall V1b \in ty_2Einteger_2Eint. \\
& ((p (ap (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V1b)) \Rightarrow ((ap c_2Efrac_2Efrac_dnm (ap c_2Efrac_2Eabs_frac \\
& (ap (ap (c_2Epair_2E_2C ty_2Einteger_2Eint ty_2Einteger_2Eint) \\
& V0a) V1b))) = V1b))))))
\end{aligned} \tag{86}$$

Assume the following.

$$(\forall V0a \in ty_2Efrac_2Efrac. ((ap (ap c_2Efrac_2Efrac_mul V0a) c_2Efrac_2Efrac_1) = V0a)) \tag{87}$$

Assume the following.

$$((ap c_2Efrac_2Efrac_ainv c_2Efrac_2Efrac_0) = c_2Efrac_2Efrac_0) \tag{88}$$

Assume the following.

$$((ap c_2Efrac_2Efrac_minv c_2Efrac_2Efrac_1) = c_2Efrac_2Efrac_1) \tag{89}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& V0x) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0x))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((\neg(p (ap (ap c_2Einteger_2Eint_lt V0x) V1y))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le \\
& V1y) V0x))))))
\end{aligned} \tag{91}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((p (ap (ap c_2Einteger_2Eint_le V0x) V1y)) \Leftrightarrow ((p (ap (ap c_2Einteger_2Eint_lt \\
& V0x) V1y)) \vee (V0x = V1y))))))
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& (p (ap (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{93}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num \\
& V0m)) (ap c_2Einteger_2Eint_of_num V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n))))))
\end{aligned} \tag{94}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
V0m)) (ap c_2Einteger_2Eint_of_num V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V0m) V1n))))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap c_2Einteger_2Eint_of_num V0m) = (ap c_2Einteger_2Eint_of_num \\
V1n)) \Leftrightarrow (V0m = V1n))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num \\
V0m)) (ap c_2Einteger_2Eint_of_num V1n)) = (ap c_2Einteger_2Eint_of_num \\
& (ap (ap c_2Earithmetric_2E_2A V0m) V1n))))))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
V0n)) (ap c_2Einteger_2Eint_of_num V1m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V0n)) (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V1m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V1m) V0n))) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V0n)) (ap c_2Einteger_2Eint_of_num \\
V1m))) \Leftrightarrow ((\neg(V0n = c_2Enum_2E0)) \vee (\neg(V1m = c_2Enum_2E0)))) \wedge ((p \\
& (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
V0n)) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
V1m)))) \Leftrightarrow False))))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint. ((\exists V1n \in ty_2Enum_2Enum. \\
& ((V0p = (ap c_2Einteger_2Eint_of_num V1n)) \wedge (\neg(V1n = c_2Enum_2E0)))) \vee \\
& ((\exists V2n \in ty_2Enum_2Enum. ((V0p = (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V2n)) \wedge (\neg(V2n = c_2Enum_2E0)))) \vee \\
& (V0p = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Einteger_2Eint (ap c_2Einteger_2Eint_of_num \\
V0n)) = V0n))
\end{aligned} \tag{100}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap\ c_2Einteger_2EABS\ (ap\ c_2Einteger_2Eint_of_num\ V0n)) = (ap\ c_2Einteger_2Eint_of_num\ V0n))) \quad (101)$$

Assume the following.

$$(\forall V0p \in ty_2Einteger_2Eint.((ap\ c_2Einteger_2EABS\ (ap\ c_2Einteger_2Eint_neg\ V0p)) = (ap\ c_2Einteger_2EABS\ V0p))) \quad (102)$$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\ & (((ap\ c_2Einteger_2Eint_of_num\ V0m) = (ap\ c_2Einteger_2Eint_of_num\ V1n)) \Leftrightarrow (V0m = V1n)))) \wedge ((\forall V2x \in ty_2Einteger_2Eint.(\forall V3y \in \\ & ty_2Einteger_2Eint.(((ap\ c_2Einteger_2Eint_neg\ V2x) = (ap\ c_2Einteger_2Eint_neg\ V3y)) \Leftrightarrow (V2x = V3y)))) \wedge (\forall V4n \in ty_2Enum_2Enum.(\forall V5m \in \\ & ty_2Enum_2Enum.(((ap\ c_2Einteger_2Eint_of_num\ V4n) = (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num\ V5m))) \Leftrightarrow \\ & ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))) \wedge (((ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num\ V4n)) = (ap\ c_2Einteger_2Eint_of_num\ V5m)) \Leftrightarrow ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0)))))))))) \end{aligned} \quad (103)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \quad (104)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\ & (\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC\ V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (105)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c_2Earithmic_2EBIT1\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
& \tag{107}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO)\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO) \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& V0n)\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))))))))) \\
& \tag{108}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ c_2Earithmic_2EZERO)\ V0n)) \Leftrightarrow \\
& True) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))))))))) \\
& \tag{109}
\end{aligned}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap\ c_2Enum_2ESUC\ V0m) = (ap\ c_2Enum_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \quad (110)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ c_2Enum_2E0)))) \quad (111)$$

Assume the following.

$$(p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ ty_2Efrac_2Efrac\ ty_2Erat_2Erat)\ c_2Erat_2Erat_equiv)\ c_2Erat_2Eabs_rat)\ c_2Erat_2Erep_rat)) \quad (112)$$

Assume the following.

$$((ap\ c_2Erat_2Erat_of_num\ c_2Enum_2E0) = (ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_0)) \quad (113)$$

Assume the following.

$$((ap\ c_2Erat_2Erat_of_num\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) = (ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_1)) \quad (114)$$

Assume the following.

$$(\forall V0x \in ty_2Efrac_2Efrac.((ap\ c_2Erat_2Eabs_rat\ (ap\ c_2Efrac_2Efrac_ainv\ (ap\ c_2Erat_2Erep_rat\ (ap\ c_2Erat_2Eabs_rat\ V0x)))) = (ap\ c_2Erat_2Eabs_rat\ (ap\ c_2Efrac_2Efrac_ainv\ V0x)))) \quad (115)$$

Assume the following.

$$((\forall V0x \in ty_2Efrac_2Efrac.(\forall V1y \in ty_2Efrac_2Efrac.((ap\ c_2Erat_2Eabs_rat\ (ap\ (ap\ c_2Efrac_2Efrac_mul\ (ap\ c_2Erat_2Erep_rat\ (ap\ c_2Erat_2Eabs_rat\ V0x))\ V1y)) = (ap\ c_2Erat_2Eabs_rat\ (ap\ (ap\ c_2Efrac_2Efrac_mul\ V0x)\ V1y)))))) \wedge (\forall V2x \in ty_2Efrac_2Efrac.(\forall V3y \in ty_2Efrac_2Efrac.((ap\ c_2Erat_2Eabs_rat\ (ap\ (ap\ c_2Efrac_2Efrac_mul\ V2x)\ (ap\ c_2Erat_2Erep_rat\ (ap\ c_2Erat_2Eabs_rat\ V3y)))) = (ap\ c_2Erat_2Eabs_rat\ (ap\ (ap\ c_2Efrac_2Efrac_mul\ V2x)\ V3y)))))) \quad (116)$$

Assume the following.

$$(\forall V0f1 \in ty_2Efrac_2Efrac.(\neg((ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0) = (ap\ c_2Efrac_2Efrac_nmr\ V0f1))) \Rightarrow ((ap\ c_2Erat_2Erat_minv\ (ap\ c_2Erat_2Eabs_rat\ V0f1)) = (ap\ c_2Erat_2Eabs_rat\ (ap\ c_2Efrac_2Efrac_minv\ V0f1)))) \quad (117)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f1 \in ty_2Efrac_2Efrac. (\forall V1f2 \in ty_2Efrac_2Efrac. \\
& ((p (ap (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Eabs_rat V0f1)) (\\
& ap c_2Erat_2Eabs_rat V1f2))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt \\
& (ap (ap c_2Einteger_2Eint_mul (ap c_2Efrac_2Efrac_nmr V0f1)) \\
& (ap c_2Efrac_2Efrac_dnm V1f2))) (ap (ap c_2Einteger_2Eint_mul \\
& (ap c_2Efrac_2Efrac_nmr V1f2)) (ap c_2Efrac_2Efrac_dnm V0f1)))))))))
\end{aligned} \tag{118}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f1 \in ty_2Efrac_2Efrac. (\forall V1f2 \in ty_2Efrac_2Efrac. \\
& ((p (ap (ap (ap c_2Erat_2Erat_leq (ap c_2Erat_2Eabs_rat V0f1)) (\\
& ap c_2Erat_2Eabs_rat V1f2))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le \\
& (ap (ap c_2Einteger_2Eint_mul (ap c_2Efrac_2Efrac_nmr V0f1)) \\
& (ap c_2Efrac_2Efrac_dnm V1f2))) (ap (ap c_2Einteger_2Eint_mul \\
& (ap c_2Efrac_2Efrac_nmr V1f2)) (ap c_2Efrac_2Efrac_dnm V0f1)))))))))
\end{aligned} \tag{119}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n1 \in ty_2Enum_2Enum. ((ap c_2Erat_2Erat_of_num V0n1) = \\
& (ap c_2Erat_2Eabs_rat (ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C \\
& ty_2Einteger_2Eint ty_2Einteger_2Eint) (ap c_2Einteger_2Eint_of_num \\
& V0n1)) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))))))
\end{aligned} \tag{120}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Enum_2Enum. (\forall V1b \in ty_2Enum_2Enum. (\\
& (p (ap (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_of_num V0a)) \\
& (ap c_2Erat_2Erat_of_num V1b))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V0a) V1b))))))
\end{aligned} \tag{121}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erat_2Erat. (\forall V1b \in ty_2Erat_2Erat. (\\
& (ap (ap c_2Erat_2Erat_add V0a) V1b) = (ap (ap c_2Erat_2Erat_add \\
& V1b) V0a))))
\end{aligned} \tag{122}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& ((ap (ap c_2Erat_2Erat_sub V0r1) V1r2) = (ap (ap c_2Erat_2Erat_add \\
& V0r1) (ap c_2Erat_2Erat_ainv V1r2))))))
\end{aligned} \tag{123}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& ((ap (ap c_2Erat_2Erat_div V0r1) V1r2) = (ap (ap c_2Erat_2Erat_mul \\
& V0r1) (ap c_2Erat_2Erat_minv V1r2))))))
\end{aligned} \tag{124}$$

Assume the following.

$$\begin{aligned} & ((ap\ c_2Erat_2Erat_ainv\ (ap\ c_2Erat_2Erat_of_num\ c_2Enum_2E0)) = \\ & \quad (ap\ c_2Erat_2Erat_of_num\ c_2Enum_2E0)) \end{aligned} \quad (125)$$

Assume the following.

$$\begin{aligned} & (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\ & ((ap\ c_2Erat_2Erat_ainv\ (ap\ (ap\ c_2Erat_2Erat_add\ V0r1)\ V1r2)) = \\ & \quad (ap\ (ap\ c_2Erat_2Erat_add\ (ap\ c_2Erat_2Erat_ainv\ V0r1))\ (ap \\ & \quad \quad c_2Erat_2Erat_ainv\ V1r2)))))) \end{aligned} \quad (126)$$

Assume the following.

$$\begin{aligned} & (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\ & (((ap\ c_2Erat_2Erat_ainv\ V0r1) = V1r2) \Leftrightarrow (V0r1 = (ap\ c_2Erat_2Erat_ainv \\ & \quad V1r2)))))) \end{aligned} \quad (127)$$

Assume the following.

$$\begin{aligned} & (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\ & (((ap\ c_2Erat_2Erat_ainv\ V0r1) = (ap\ c_2Erat_2Erat_ainv\ V1r2)) \Leftrightarrow \\ & \quad (V0r1 = V1r2)))))) \end{aligned} \quad (128)$$

Assume the following.

$$\begin{aligned} & (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\ & ((p\ (ap\ (ap\ c_2Erat_2Erat_les\ V0r1)\ V1r2)) \Rightarrow (\neg (p\ (ap\ (ap\ c_2Erat_2Erat_les \\ & \quad V1r2)\ V0r1)))))) \end{aligned} \quad (129)$$

Assume the following.

$$\begin{aligned} & (\forall V0r1 \in ty_2Erat_2Erat. (p\ (ap\ (ap\ c_2Erat_2Erat_leq\ V0r1) \\ & \quad V0r1))) \end{aligned} \quad (130)$$

Assume the following.

$$\begin{aligned} & (p\ (ap\ (ap\ c_2Erat_2Erat_les\ (ap\ c_2Erat_2Erat_of_num\ c_2Enum_2E0)) \\ & \quad (ap\ c_2Erat_2Erat_of_num\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap \\ & \quad \quad c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \end{aligned} \quad (131)$$

Assume the following.

$$\begin{aligned} & (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\ & ((\neg (p\ (ap\ (ap\ c_2Erat_2Erat_les\ V1r2)\ V0r1))) \Leftrightarrow (p\ (ap\ (ap\ c_2Erat_2Erat_leq \\ & \quad V0r1)\ V1r2)))))) \end{aligned} \quad (132)$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erat_2Erat. (\forall V1b \in ty_2Erat_2Erat. (\\
& \quad \forall V2c \in ty_2Erat_2Erat. (((p (ap (ap c_2Erat_2Erat_les V0a) \\
& \quad V1b)) \wedge (p (ap (ap c_2Erat_2Erat_leq V1b) V2c))) \Rightarrow (p (ap (ap c_2Erat_2Erat_les \\
& \quad \quad V0a) V2c))))))
\end{aligned} \tag{133}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erat_2Erat. (\forall V1b \in ty_2Erat_2Erat. (\\
& \quad \forall V2c \in ty_2Erat_2Erat. (((p (ap (ap c_2Erat_2Erat_leq V0a) \\
& \quad V1b)) \wedge (p (ap (ap c_2Erat_2Erat_les V1b) V2c))) \Rightarrow (p (ap (ap c_2Erat_2Erat_les \\
& \quad \quad V0a) V2c))))))
\end{aligned} \tag{134}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& ((p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) \\
& \quad V0r1)) \Rightarrow ((p (ap (ap c_2Erat_2Erat_leq (ap c_2Erat_2Erat_of_num \\
& \quad c_2Enum_2E0)) V1r2)) \Rightarrow (p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_of_num \\
& \quad \quad c_2Enum_2E0)) (ap (ap c_2Erat_2Erat_add V0r1) V1r2))))))
\end{aligned} \tag{135}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& (\forall V2r3 \in ty_2Erat_2Erat. ((V0r1 = (ap (ap c_2Erat_2Erat_sub \\
& \quad V1r2) V2r3)) \Leftrightarrow ((ap (ap c_2Erat_2Erat_add V0r1) V2r3) = V1r2))))))
\end{aligned} \tag{136}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& (\forall V2r3 \in ty_2Erat_2Erat. (((ap (ap c_2Erat_2Erat_add V0r1) \\
& \quad V2r3) = (ap (ap c_2Erat_2Erat_add V1r2) V2r3)) \Leftrightarrow (V0r1 = V1r2))))))
\end{aligned} \tag{137}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& ((p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_ainv V0r1)) \\
& \quad (ap c_2Erat_2Erat_ainv V1r2))) \Leftrightarrow (p (ap (ap c_2Erat_2Erat_les \\
& \quad \quad V1r2) V0r1))))))
\end{aligned} \tag{138}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& ((p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_ainv V0r1)) \\
& \quad V1r2)) \Leftrightarrow (p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_ainv \\
& \quad \quad V1r2)) V0r1))))))
\end{aligned} \tag{139}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Erat_2Erat. (\forall V1q \in ty_2Erat_2Erat. (\\
& ((p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) \\
& (ap (ap c_2Erat_2Erat_mul V0p) V1q))) \Leftrightarrow (((p (ap (ap c_2Erat_2Erat_les \\
& (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) V0p)) \wedge (p (ap (ap c_2Erat_2Erat_les \\
& (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) V1q))) \vee ((p (ap (ap c_2Erat_2Erat_les \\
& V0p) (ap c_2Erat_2Erat_of_num c_2Enum_2E0))) \wedge (p (ap (ap c_2Erat_2Erat_les \\
& V1q) (ap c_2Erat_2Erat_of_num c_2Enum_2E0)))))) \wedge ((p (ap (ap \\
& c_2Erat_2Erat_les (ap (ap c_2Erat_2Erat_mul V0p) V1q)) (ap c_2Erat_2Erat_of_num \\
& c_2Enum_2E0))) \Leftrightarrow (((p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_of_num \\
& c_2Enum_2E0)) V0p)) \wedge (p (ap (ap c_2Erat_2Erat_les V1q) (ap c_2Erat_2Erat_of_num \\
& c_2Enum_2E0)))) \vee ((p (ap (ap c_2Erat_2Erat_les V0p) (ap c_2Erat_2Erat_of_num \\
& c_2Enum_2E0))) \wedge (p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_of_num \\
& c_2Enum_2E0)) V1q)))))))))
\end{aligned} \tag{140}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& (((V0r1 = (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) \vee (V1r2 = (ap \\
& c_2Erat_2Erat_of_num c_2Enum_2E0))) \Leftrightarrow ((ap (ap c_2Erat_2Erat_mul \\
& V0r1) V1r2) = (ap c_2Erat_2Erat_of_num c_2Enum_2E0))))))
\end{aligned} \tag{141}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in A_27a. (((ap c_2Erat_2Erat_of_num \\
& c_2Enum_2E0) = c_2Erat_2Erat_0) \wedge (\forall V1n \in ty_2Enum_2Enum. \\
& ((ap c_2Erat_2Erat_of_num (ap c_2Enum_2ESUC V1n)) = (ap (ap c_2Erat_2Erat_add \\
& (ap c_2Erat_2Erat_of_num V1n)) c_2Erat_2Erat_1))))))
\end{aligned} \tag{142}$$

Assume the following.

$$(c_2Erat_2Erat_0 = (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) \tag{143}$$

Assume the following.

$$(c_2Erat_2Erat_1 = (ap c_2Erat_2Erat_of_num (ap c_2Earithmetic_2ENUMERAL \\
(ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \tag{144}$$

Assume the following.

$$\begin{aligned}
& (\forall V0i1 \in ty_2Einteger_2Eint. (\forall V1i2 \in ty_2Einteger_2Eint. \\
& (((ap c_2Erat_2Erat_of_int V0i1) = (ap c_2Erat_2Erat_of_int \\
& V1i2)) \Leftrightarrow (V0i1 = V1i2))))
\end{aligned} \tag{145}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. ((ap c_2Erat_2Erat_of_int (ap \\
& c_2Einteger_2Eint_of_num V0x)) = (ap c_2Erat_2Erat_of_num \\
& V0x)))
\end{aligned} \tag{146}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap (ap c_2Erat_2Erat_mul (ap c_2Erat_2Erat_of_int V0x)) \\
& (ap c_2Erat_2Erat_of_int V1y)) = (ap c_2Erat_2Erat_of_int \\
& (ap (ap c_2Einteger_2Eint_mul V0x) V1y))))))
\end{aligned} \tag{147}$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty_2Einteger_2Eint. (\forall V1j \in ty_2Einteger_2Eint. \\
& ((p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_of_int V0i)) \\
& (ap c_2Erat_2Erat_of_int V1j))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt \\
& V0i) V1j))))))
\end{aligned} \tag{148}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r \in ty_2Erat_2Erat. ((V0r = (ap (ap c_2Erat_2Erat_div \\
& (ap c_2Erat_2Erat_of_int (ap c_2Erat_2ERATN V0r)) (ap c_2Erat_2Erat_of_num \\
& (ap c_2Erat_2ERATD V0r)))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) \\
& (ap c_2Erat_2ERATD V0r))) \wedge (((ap c_2Erat_2ERATN V0r) = (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) \Rightarrow ((ap c_2Erat_2ERATD V0r) = (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge (\forall V1n_27 \in \\
& ty_2Einteger_2Eint. (\forall V2d_27 \in ty_2Enum_2Enum. ((V0r = \\
& (ap (ap c_2Erat_2Erat_div (ap c_2Erat_2Erat_of_int V1n_27)) \\
& (ap c_2Erat_2Erat_of_num V2d_27))) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V2d_27))) \Rightarrow (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2EABS \\
& (ap c_2Erat_2ERATN V0r)) (ap c_2Einteger_2EABS V1n_27))))))))))
\end{aligned} \tag{149}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r \in ty_2Erat_2Erat. (\forall V1n_27 \in ty_2Einteger_2Eint. \\
& (\forall V2d_27 \in ty_2Enum_2Enum. (((V0r = (ap (ap c_2Erat_2Erat_div \\
& (ap c_2Erat_2Erat_of_int V1n_27)) (ap c_2Erat_2Erat_of_num \\
& V2d_27))) \wedge (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V2d_27))) \Rightarrow \\
& (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2EABS (ap c_2Erat_2ERATN \\
& V0r)) (ap c_2Einteger_2EABS V1n_27))))))))))
\end{aligned} \tag{150}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r \in ty_2Erat_2Erat. ((ap (ap c_2Erat_2Erat_mul V0r) \\
& (ap c_2Erat_2Erat_of_num (ap c_2Erat_2ERATD V0r))) = (ap c_2Erat_2Erat_of_int \\
& (ap c_2Erat_2ERATN V0r))))
\end{aligned} \tag{151}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{152}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{153}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (154)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (155)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (156)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (157)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (158)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (159)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (160)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (161)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (162)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (163)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (164)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (165)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (166)$$

Theorem 1

$$\begin{aligned} & (\forall V0x \in ty_2Erat_2Erat.(((p (ap (ap c_2Einteger_2Eint_lt \\ & (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap c_2Erat_2ERATN \\ & V0x))) \Leftrightarrow (p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_of_num \\ c_2Enum_2E0)) V0x))) \wedge (((p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num \\ c_2Enum_2E0)) (ap c_2Erat_2ERATN V0x))) \Leftrightarrow (p (ap (ap c_2Erat_2Erat_leq \\ (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) V0x))) \wedge (((p (ap (ap \\ c_2Einteger_2Eint_lt (ap c_2Erat_2ERATN V0x)) (ap c_2Einteger_2Eint_of_num \\ c_2Enum_2E0))) \Leftrightarrow (p (ap (ap c_2Erat_2Erat_les V0x) (ap c_2Erat_2Erat_of_num \\ c_2Enum_2E0)))) \wedge ((p (ap (ap c_2Einteger_2Eint_le (ap c_2Erat_2ERATN \\ V0x)) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow (p (ap (\\ ap c_2Erat_2Erat_leq V0x) (ap c_2Erat_2Erat_of_num c_2Enum_2E0)))))))))) \end{aligned}$$