

thm\_2Erat\_2ERAT\_\_0LES\_\_0LEQ\_\_ADD  
(TMUnzbnh4soGFaiS14K4zejDsS3Lncy9hKX)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o(x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Erat\_2Erat : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erat\_2Erat \tag{1}$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{2}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 3** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 4** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{5}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{6}$$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (c\_2Enum\_2ESUC\_REP\ m))$

Let  $c\_2Earithmic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 7** We define  $c\_2Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmic\_2E\_2B$

**Definition 8** We define  $c\_2Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \quad (8)$$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $ty\_2Efrac\_2Efrac : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \quad (10)$$

Let  $c\_2Erat\_2Erep\_rat\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erat\_2Erep\_rat\_CLASS \in ((2^{ty\_2Efrac\_2Efrac})^{ty\_2Erat\_2Erat}) \quad (11)$$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 10** We define  $c\_2Erat\_2Erep\_rat$  to be  $\lambda V0a \in ty\_2Erat\_2Erat.(ap (c\_2Emin\_2E\_40\ ty\_2Efrac\_2Efrac$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (12)$$

Let  $c\_2Efrac\_2Erep\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Erep\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \quad (13)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Epair\_2ESND\ A.27a\ A.27b \in (A.27b^{(ty\_2Epair\_2Eprod\ A.27a\ A.27b)}) \quad (14)$$

**Definition 11** We define  $c\_2Efrac\_2Efrac\_dnm$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap (c\_2Epair\_2ESND\ t$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Epair\_2EFST\ A.27a\ A.27b \in (A.27a^{(ty\_2Epair\_2Eprod\ A.27a\ A.27b)}) \quad (15)$$

**Definition 12** We define  $c\_Efrac\_Efrac\_nmr$  to be  $\lambda V0f \in ty\_Efrac\_Efrac.(ap (c\_Epair\_EFST ty\_Eint\_Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_REP\_CLASS \in ((2^{(ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)})^{ty\_Einteger\_Eint}) \quad (16)$$

**Definition 13** We define  $c\_Einteger\_Eint\_REP$  to be  $\lambda V0a \in ty\_Einteger\_Eint.(ap (c\_Emin\_E.40 (ty\_Einteger\_Eint\_neg : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_neg \in ((ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)^{(ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)}) \quad (17)$$

Let  $c\_Einteger\_Eint\_eq : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_eq \in ((2^{(ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)})^{(ty\_Epair\_Eprod ty\_Eenum\_Eenum)}) \quad (18)$$

Let  $c\_Einteger\_Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_ABS\_CLASS \in (ty\_Einteger\_Eint)^{(2^{(ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)})} \quad (19)$$

**Definition 14** We define  $c\_Einteger\_Eint\_ABS$  to be  $\lambda V0r \in (ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)$

**Definition 15** We define  $c\_Einteger\_Eint\_neg$  to be  $\lambda V0T1 \in ty\_Einteger\_Eint.(ap c\_Einteger\_Eint\_neg$

**Definition 16** We define  $c\_Emin\_E.3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 17** We define  $c\_Ebool\_E.2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_E.21 2) (\lambda V2t$

Let  $c\_Epair\_EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_Epair\_EABS\_prod A\_27a A\_27b \in ((ty\_Epair\_Eprod A\_27a A\_27b)^{(2^{A\_27b} A\_27a})} \quad (20)$$

**Definition 18** We define  $c\_Epair\_E.2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_Epair\_EABS\_prod$

Let  $c\_Efrac\_Eabs\_frac : \iota$  be given. Assume the following.

$$c\_Efrac\_Eabs\_frac \in (ty\_Efrac\_Efrac)^{(ty\_Epair\_Eprod ty\_Einteger\_Eint ty\_Einteger\_Eint)} \quad (21)$$

**Definition 19** We define  $c\_Efrac\_Efrac\_ainv$  to be  $\lambda V0f1 \in ty\_Efrac\_Efrac.(ap c\_Efrac\_Eabs\_frac$

Let  $c\_Einteger\_Eint\_mul : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_mul \in (((ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)^{(ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)})^{(ty\_Epair\_Eprod ty\_Eenum\_Eenum ty\_Eenum\_Eenum)}) \quad (22)$$

**Definition 20** We define `c_2Einteger_2Eint_mul` to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$ . Let `c_2Einteger_2Etint_add` :  $\iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))$$
(23)

**Definition 21** We define `c_2Einteger_2Eint_add` to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

**Definition 22** We define `c_2Efrac_2Efrac_add` to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

**Definition 23** We define `c_2Efrac_2Efrac_sub` to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

**Definition 24** We define `c_2Erat_2Erat_equiv` to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

Let `c_2Erat_2Eabs_rat_CLASS` :  $\iota$  be given. Assume the following.

$$c\_2Erat\_2Eabs\_rat\_CLASS \in (ty\_2Erat\_2Erat^{(2^{ty\_2Efrac\_2Efrac})})$$
(24)

**Definition 25** We define `c_2Erat_2Eabs_rat` to be  $\lambda V0r \in ty\_2Efrac\_2Efrac.(ap\ c\_2Erat\_2Eabs\_rat\_CLASS)$

**Definition 26** We define `c_2Erat_2Erat_sub` to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap\ c\_2Erat\_2Erat\_sub)$

Let `c_2Einteger_2Etint_lt` :  $\iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)))$$
(25)

**Definition 27** We define `c_2Einteger_2Eint_lt` to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

**Definition 28** We define `c_2Ebool_2EF` to be  $(ap\ (c\_2Ebool\_2E_21\ 2))\ (\lambda V0t \in 2.V0t)$ .

**Definition 29** We define `c_2Ebool_2ECOND` to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap\ c\_2Ebool\_2ECOND))))$

**Definition 30** We define `c_2EintExtension_2ESGN` to be  $\lambda V0x \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND))))$

**Definition 31** We define `c_2Efrac_2Efrac_sgn` to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2EintExtension\_2ESGN)$

**Definition 32** We define `c_2Erat_2Erat_sgn` to be  $\lambda V0r \in ty\_2Erat\_2Erat.(ap\ c\_2Efrac\_2Efrac\_sgn)$

**Definition 33** We define `c_2Erat_2Erat_les` to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap\ (ap\ c\_2Erat\_2Erat\_sgn))$

**Definition 34** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E_21\ 2))\ (\lambda V2t \in 2.V0t))))$

**Definition 35** We define `c_2Erat_2Erat_leq` to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap\ (ap\ c\_2Erat\_2Erat\_les))$

**Definition 36** We define `c_2Erat_2Erat_add` to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap\ (ap\ c\_2Erat\_2Erat\_add))$

**Definition 37** We define `c_2Efrac_2Efrac_1` to be  $(ap\ c\_2Efrac\_2Eabs\_frac)\ (ap\ (ap\ (c\_2Epair\_2E_2C\ ty\_2Enum\_2Enum)))$

**Definition 38** We define `c_2Erat_2Erat_1` to be  $(ap\ c\_2Erat\_2Eabs\_rat\ c\_2Efrac\_2Efrac\_1)$ .

**Definition 39** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 40** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 41** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A\_27a))$

Let  $c\_2Earithmetic\_2Enum\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Earithmetic\_2Enum\_CASE A\_27a \in \left( (A\_27a^{(A\_27a^{ty\_2Enum\_2Enum})})^{A\_27a} \right)^{ty\_2Enum\_2Enum} \quad (26)$$

**Definition 42** We define  $c\_2Efrac\_2Efrac\_0$  to be  $(ap c\_2Efrac\_2Eabs\_frac (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum))$

**Definition 43** We define  $c\_2Erat\_2Erat\_0$  to be  $(ap c\_2Erat\_2Eabs\_rat c\_2Efrac\_2Efrac\_0)$ .

**Definition 44** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

**Definition 45** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 46** We define  $c\_2Erelation\_2EWF$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E\_21$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ebool\_2EARB A\_27a \in A\_27a \quad (27)$$

**Definition 47** We define  $c\_2Erelation\_2ERESTRICT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

**Definition 48** We define  $c\_2Erelation\_2ETC$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

**Definition 49** We define  $c\_2Erelation\_2Eapprox$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 50** We define  $c\_2Erelation\_2Ethe\_fun$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 51** We define  $c\_2Erelation\_2EWFREC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 52** We define  $c\_2Erat\_2Erat\_of\_num$  to be  $(ap (ap (c\_2Erelation\_2EWFREC ty\_2Enum\_2Enum$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee (\neg(p \ V0t)))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\ & (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\ & (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p \ V0t)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ( \\ & (p \ V1B) \vee (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \end{aligned} \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))))) \quad (42)$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat. ((ap (ap c\_2Erat\_2Erat\_add V0a) (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) = V0a)) \quad (43)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. ((p (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) V0r1)) \Rightarrow ((p (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) V1r2)) \Rightarrow (p (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) (ap (ap c\_2Erat\_2Erat\_add V0r1) V1r2))))))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee \neg(p V1q))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee \neg(p V0p))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee \neg(p V0p)))))))) \quad (50)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee \neg(p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (( \\
& \neg(p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p))))))
\end{aligned} \tag{53}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{54}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \tag{55}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \tag{58}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0r1 \in ty\_2Erat\_2Erat. (\forall V1r2 \in ty\_2Erat\_2Erat. \\
& ((p (ap (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) \\
& V0r1)) \Rightarrow ((p (ap (ap (ap c\_2Erat\_2Erat\_leq (ap c\_2Erat\_2Erat\_of\_num \\
& c\_2Enum\_2E0)) V1r2)) \Rightarrow (p (ap (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap c\_2Erat\_2Erat\_add V0r1) V1r2)))))))))
\end{aligned}$$