

thm_2Erat_2ERAT__0LES__0LES__ADD
(TMbN7USMKPYiwRK3bpzUk98Gc7XMycYAzdZ)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \tag{3}$$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \tag{4}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2ESND\ A.27a\ A.27b \in (A.27b)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \tag{5}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (c_2Ebool_2E_21\ P)) (c_2Ebool_2E_21\ P)))$

Definition 4 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND\ ty_2Efrac_2Efrac\ f))$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod ty_2Einteger_2Eint ty_2Einteger_2Eint)}) \quad (7)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (8)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (9)$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 9 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty$

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (10)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (11)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})} \quad (12)$$

Definition 10 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$

Definition 11 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Let $c_2Einteger_2Eint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (13)$$

Definition 12 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$.
Let $ty_2Erat_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \quad (14)$$

Let $c_2Erat_2Erep_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (15)$$

Definition 13 We define $c_2Erat_2Erep_rat$ to be $\lambda V0a \in ty_2Erat_2Erat.(ap\ (c_2Emin_2E40\ ty_2Efrac$.

Let $c_2Epair_2Efst : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2Efst \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (16)$$

Definition 14 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2Efst\ ty$.

Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum\ ty_2Eenum_2Eenum)})^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum)}) \quad (17)$$

Definition 15 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$.

Definition 16 We define $c_2Einteger_2Eint_gt$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$.

Definition 17 We define $c_2Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$.

Definition 18 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$.

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat^{(2^{ty_2Efrac_2Efrac})}) \quad (18)$$

Definition 19 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap\ c_2Erat_2Eabs_rat_CLASS$.

Definition 20 We define $c_2Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$.

Definition 21 We define $c_2Erat_2Erat_nmr$ to be $\lambda V0r \in ty_2Erat_2Erat.(ap\ c_2Efrac_2Efrac_nmr\ (ap$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Eenum_2Eenum}) \quad (19)$$

Let $c_2Eenum_2Ezero_rep : \iota$ be given. Assume the following.

$$c_2Eenum_2Ezero_rep \in omega \quad (20)$$

Let $c_2Eenum_2Eabs_num : \iota$ be given. Assume the following.

$$c_2Eenum_2Eabs_num \in (ty_2Eenum_2Eenum^{omega}) \quad (21)$$

Definition 22 We define c_Enum_2E0 to be $(ap\ c_Enum_2EABS_num\ c_Enum_2EZERO_REP)$.

Definition 23 We define $c_Earithmetic_2EZERO$ to be c_Enum_2E0 .

Let $c_Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (22)$$

Let $c_Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_Enum_2ESUC_REP \in (\omega^{omega}) \quad (23)$$

Definition 24 We define c_Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_Enum_2EABS_num$

Let $c_Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (24)$$

Definition 25 We define $c_Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_Earithmetic$

Definition 26 We define $c_Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 27 We define $c_Efrac_2Efrac_1$ to be $(ap\ c_Efrac_2Eabs_frac\ (ap\ (ap\ (c_Epair_2E_2C\ ty_2$

Definition 28 We define $c_Erat_2Erat_1$ to be $(ap\ c_Erat_2Eabs_rat\ c_Efrac_2Efrac_1)$.

Definition 29 We define $c_Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 30 We define $c_Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27$

Definition 31 We define $c_Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A$

Let $c_Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Earithmetic_2Enum_CASE\ A_27a \in \left(\left((A_27a^{(A_27a^{ty_2Enum_2Enum})})^{A_27a} \right)^{ty_2Enum_2Enum} \right) \quad (25)$$

Definition 32 We define $c_Efrac_2Efrac_0$ to be $(ap\ c_Efrac_2Eabs_frac\ (ap\ (ap\ (c_Epair_2E_2C\ ty_2$

Definition 33 We define $c_Erat_2Erat_0$ to be $(ap\ c_Erat_2Eabs_rat\ c_Efrac_2Efrac_0)$.

Definition 34 We define c_Ebool_2EF to be $(ap\ (c_Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 35 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_Emin_2E_3D_3D_3E\ V0t)\ c_Ebool_2E$

Definition 36 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_Emin_2E_40$

Definition 37 We define $c_Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ (c_Ebool_2E_21$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (26)$$

Definition 38 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 39 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1f \in$

Definition 40 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1a \in A_27a. \lambda V2b \in$

Definition 41 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in$

Definition 42 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in$

Definition 43 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in$

Definition 44 We define $c_2Erat_2Erat_of_num$ to be $(ap\ (ap\ (c_2Erelation_2EWFREC\ ty_2Enum_2Enum$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Einteger_2Etint_neg \in & ((ty_2Epair_2Eprod\ ty_2Enum_2Enum \\ & ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \end{aligned} \quad (27)$$

Definition 45 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. (ap\ c_2Einteger_2Eint.$

Definition 46 We define $c_2Efrac_2Efrac_ainv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. (ap\ c_2Efrac_2Eabs_fn$

Definition 47 We define $c_2Efrac_2Efrac_sub$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. \lambda V1f2 \in ty_2Efrac_2Efrac.$

Definition 48 We define $c_2Erat_2Erat_sub$ to be $\lambda V0r1 \in ty_2Erat_2Erat. \lambda V1r2 \in ty_2Erat_2Erat. (ap$

Definition 49 We define $c_2EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint. (ap\ (ap\ (ap\ (c_2Ebool$

Definition 50 We define $c_2Efrac_2Efrac_sgn$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. (ap\ c_2EintExtension_2ES$

Definition 51 We define $c_2Erat_2Erat_sgn$ to be $\lambda V0r \in ty_2Erat_2Erat. (ap\ c_2Efrac_2Efrac_sgn\ (ap\ c_2$

Definition 52 We define $c_2Erat_2Erat_les$ to be $\lambda V0r1 \in ty_2Erat_2Erat. \lambda V1r2 \in ty_2Erat_2Erat. (ap\ ($

Definition 53 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3)))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (33)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False) \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A_27a.(p (ap V1Q V4x)))))))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ((p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))))) \quad (40)$$

Assume the following.

$$(\forall V0f \in ty_2Efrac_2Efrac. ((ap\ c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint) (ap\ c_2Efrac_2Efrac_nmr\ V0f)) (ap\ c_2Efrac_2Efrac_dnm\ V0f))) = V0f)) \quad (41)$$

Assume the following.

$$(\forall V0f \in ty_2Efrac_2Efrac. (p (ap (ap\ c_2Einteger_2Eint_lt (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) (ap\ c_2Efrac_2Efrac_dnm\ V0f)))) \quad (42)$$

Assume the following.

$$(\forall V0a \in ty_2Einteger_2Eint. (\forall V1b \in ty_2Einteger_2Eint. ((p (ap (ap\ c_2Einteger_2Eint_lt (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) V1b)) \Rightarrow ((ap\ c_2Efrac_2Efrac_nmr (ap\ c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint) V0a) V1b))) = V0a)))) \quad (43)$$

Assume the following.

$$(\forall V0a1 \in ty_2Einteger_2Eint. (\forall V1b1 \in ty_2Einteger_2Eint. (\forall V2a2 \in ty_2Einteger_2Eint. (\forall V3b2 \in ty_2Einteger_2Eint. ((p (ap (ap\ c_2Einteger_2Eint_lt (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) V1b1)) \Rightarrow ((p (ap (ap\ c_2Einteger_2Eint_lt (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) V3b2)) \Rightarrow ((ap (ap\ c_2Efrac_2Efrac_add (ap\ c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint) V0a1) V1b1)) (ap\ c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint) V2a2) V3b2))) = (ap\ c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint) (ap (ap\ c_2Einteger_2Eint_add (ap (ap\ c_2Einteger_2Eint_mul V0a1) V3b2)) (ap (ap\ c_2Einteger_2Eint_mul V2a2) V1b1))) (ap (ap\ c_2Einteger_2Eint_mul V1b1) V3b2)))))))))) \quad (44)$$

Assume the following.

$$(\forall V0a \in ty_2Einteger_2Eint. (\forall V1b \in ty_2Einteger_2Eint. ((p (ap (ap\ c_2Einteger_2Eint_lt (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) V0a)) \Rightarrow ((p (ap (ap\ c_2Einteger_2Eint_lt (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) V1b)) \Rightarrow (p (ap (ap\ c_2Einteger_2Eint_lt (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) (ap (ap\ c_2Einteger_2Eint_mul V0a) V1b)))))) \quad (45)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (((p (ap (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add V0x V1y))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint. (\forall V1q \in ty_2Einteger_2Eint. \\
& (((p (ap (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_mul V0p V1q))) \Leftrightarrow (((\\
& p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V0p)) \wedge (p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V1q))) \vee ((p (ap (ap c_2Einteger_2Eint_lt V0p) (\\
& ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \wedge (p (ap (ap c_2Einteger_2Eint_lt \\
& V1q) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)))))) \wedge ((p (\\
& ap (ap c_2Einteger_2Eint_lt (ap (ap c_2Einteger_2Eint_mul V0p \\
& V1q)) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow (((p (ap \\
& (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \\
& V0p)) \wedge (p (ap (ap c_2Einteger_2Eint_lt V1q) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)))) \vee ((p (ap (ap c_2Einteger_2Eint_lt V0p) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))) \wedge (p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V1q))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r \in ty_2Erat_2Erat. ((ap c_2Erat_2Eabs_rat (ap c_2Erat_2Erep_rat \\
& V0r)) = V0r))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f1 \in ty_2Efrac_2Efrac. ((p (ap (ap c_2Einteger_2Eint_gt \\
& (ap c_2Efrac_2Efrac_nmer (ap c_2Erat_2Erep_rat (ap c_2Erat_2Eabs_rat \\
& V0f1)))) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow (p (\\
& ap c_2Einteger_2Eint_gt (ap c_2Efrac_2Efrac_nmer V0f1)) \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f1 \in ty_2Efrac_2Efrac. (\forall V1f2 \in ty_2Efrac_2Efrac. \\
& ((ap (ap c_2Erat_2Erat_add (ap c_2Erat_2Eabs_rat V0f1)) (ap \\
& c_2Erat_2Eabs_rat V1f2)) = (ap c_2Erat_2Eabs_rat (ap (ap c_2Efrac_2Efrac_add \\
& V0f1 V1f2))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. ((p (ap (ap c_2Erat_2Erat_les \\
& (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) V0r1)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap c_2Erat_2Erat_nmr \\
& V0r1))))))
\end{aligned} \tag{51}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{52}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{55}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))))) \quad (60)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q)) \vee \neg(p \ V0p)))))) \quad (61)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& ((p \ (ap \ (ap \ c_2Erat_2Erat_les \ (ap \ c_2Erat_2Erat_of_num \ c_2Enum_2E0)) \\
& V0r1)) \Rightarrow ((p \ (ap \ (ap \ c_2Erat_2Erat_les \ (ap \ c_2Erat_2Erat_of_num \\
& c_2Enum_2E0)) \ V1r2)) \Rightarrow (p \ (ap \ (ap \ c_2Erat_2Erat_les \ (ap \ c_2Erat_2Erat_of_num \\
& c_2Enum_2E0)) \ (ap \ (ap \ c_2Erat_2Erat_add \ V0r1 \ V1r2)))))))))
\end{aligned}$$