

thm_2Erat_2ERAT__ADD__ASSOC (TMG- wKipnHvXFRqPap4Q3WCULFwnf5ABmaXW)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

Let `ty_2Efrac_2Efrac` : ι be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \tag{1}$$

Let `ty_2Erat_2Erat` : ι be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \tag{2}$$

Let `c_2Erat_2Erep__rat__CLASS` : ι be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \tag{3}$$

Definition 3 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V 1x \in 2.V 1x)) (\lambda V 0x \in 2.V 0x)))$

Definition 5 We define `c_2Erat_2Erep__rat` to be $\lambda V 0a \in ty_2Erat_2Erat. (ap (c_2Emin_2E_40 ty_2Efrac_2Efrac))$

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{4}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{5}$$

Let `c_2Efrac_2Erep__frac` : ι be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \tag{6}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c.2Epair_2ESND \\ A.27a\ A.27b \in (A.27b)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \end{aligned} \quad (7)$$

Definition 6 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2ESND\ ty_2Enum_2Enum\ f))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (8)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint_REP_CLASS}) \quad (9)$$

Definition 7 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E40\ ty_2Enum_2Enum\ a))$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{c_2Einteger_2Etint_mul})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (10)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (11)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (12)$$

Definition 8 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 9 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c.2Epair_2EFST \\ A.27a\ A.27b \in (A.27a)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \end{aligned} \quad (13)$$

Definition 10 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2EFST\ ty_2Enum_2Enum\ f))$

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{c_2Einteger_2Etint_add})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (14)$$

Definition 11 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 12 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21 2) (\lambda V2t \in 2.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (15)$$

Definition 14 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod$

Let $c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod ty_2Einteger_2Eint ty_2Einteger_2Eint)}) \quad (16)$$

Definition 15 We define $c_2Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 16 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat^{(2^{ty_2Efrac_2Efrac})}) \quad (17)$$

Definition 17 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap c_2Erat_2Eabs_rat_CLASS$

Definition 18 We define $c_2Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\begin{aligned} (\forall V0a \in ty_2Efrac_2Efrac.(\forall V1b \in ty_2Efrac_2Efrac. \\ (\forall V2c \in ty_2Efrac_2Efrac.((ap (ap c_2Efrac_2Efrac_add \\ V0a) (ap (ap c_2Efrac_2Efrac_add V1b) V2c)) = (ap (ap c_2Efrac_2Efrac_add \\ (ap (ap c_2Efrac_2Efrac_add V0a) V1b)) V2c)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty_2Efrac_2Efrac. (\forall V1y \in ty_2Efrac_2Efrac. \\
& ((ap\ c_2Erat_2Eabs_rat\ (ap\ (ap\ c_2Efrac_2Efrac_add\ (ap\ c_2Erat_2Erep_rat \\
& (ap\ c_2Erat_2Eabs_rat\ V0x)))\ V1y))) = (ap\ c_2Erat_2Eabs_rat\ (\\
& ap\ (ap\ c_2Efrac_2Efrac_add\ V0x\ V1y)))))) \wedge (\forall V2x \in ty_2Efrac_2Efrac. \\
& (\forall V3y \in ty_2Efrac_2Efrac. ((ap\ c_2Erat_2Eabs_rat\ (ap\ (\\
& ap\ c_2Efrac_2Efrac_add\ V2x)\ (ap\ c_2Erat_2Erep_rat\ (ap\ c_2Erat_2Eabs_rat \\
& V3y)))))) = (ap\ c_2Erat_2Eabs_rat\ (ap\ (ap\ c_2Efrac_2Efrac_add \\
& V2x\ V3y))))))
\end{aligned} \tag{22}$$

Theorem 1

$$\begin{aligned}
& (\forall V0a \in ty_2Erat_2Erat. (\forall V1b \in ty_2Erat_2Erat. (\\
& \forall V2c \in ty_2Erat_2Erat. ((ap\ (ap\ c_2Erat_2Erat_add\ V0a) \\
& (ap\ (ap\ c_2Erat_2Erat_add\ V1b)\ V2c))) = (ap\ (ap\ c_2Erat_2Erat_add \\
& (ap\ (ap\ c_2Erat_2Erat_add\ V0a)\ V1b))\ V2c))))))
\end{aligned}$$