thm_2Erat_2ERAT__AINV__SUB (TMZibrzH5My2Kyxf2BqqZ1FAEtpEmk4b1Ny)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o \ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap \ (ap \ (\text{c\_2Emin\_2E\_3D} \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1$

Let $ty\_2Efrac\_2Efrac : \iota$ be given. Assume the following.

$$nonempty \ ty\_2Efrac\_2Efrac \tag{1}$$

Let $ty\_2Erat\_2Erat : \iota$ be given. Assume the following.

$$nonempty \ ty\_2Erat\_2Erat \tag{2}$$

Let $c\_2Erat\_2Erep\_\_rat\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Erep\_\_rat\_\_CLASS \in ((2^{ty\_2Efrac\_2Efrac})^{ty\_2Erat\_2Erat}) \tag{3}$$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if} \ (\exists x \in A.p \ (ap \ P \ x)) \ \textbf{then} \ (the \ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \ (ap \ (\text{c\_2Emin\_2E\_3D} \ (2^{A\_27}$

Definition 5 *We define* c_2Erat_2Erep__rat *to be* $\lambda V0a \in ty\_2Erat\_2Erat.(ap \ (\text{c\_2Emin\_2E\_40} \ ty\_2Efrac\_2$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty \ ty\_2Einteger\_2Eint \tag{4}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty\_2Epair\_2Eprod$$
$$A0 \ A1)$$
$$\tag{5}$$

Let $c\_2Efrac\_2Erep\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Erep\_\_frac \in ((ty\_2Epair\_2Eprod \ ty\_2Einteger\_2Eint$$
$$ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{6}$$

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Let $c\_2Epair\_2ESND : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2ESND$$
$$A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$$

$$(7)$$

Definition 6 *We define* c_2Efrac_2Efrac__dnm *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ ty\_$

Let $c\_2Epair\_2EFST : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EFST$$
$$A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$$

$$(8)$$

Definition 7 *We define* c_2Efrac_2Efrac__nmr *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty\_$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (9)$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2}$$

$$(10)$$

Definition 8 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ ($c_2Emin_2E_40 *(ty_*

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})$$

$$(11)$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum}$$

$$(12)$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}$$

$$(13)$$

Definition 9 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 10 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Definition 11 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P\Rightarrow p\ Q)$
of type ι.

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod$$
$$A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{14}$$

Definition 13 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Efrac\_2Eabs\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Eabs\_\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)}) \tag{15}$$

Definition 14 *We define* c_2Efrac_2Efrac__ainv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_\_fr$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{16}$$

Definition 15 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteg$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{17}$$

Definition 16 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteg$

Definition 17 *We define* c_2Efrac_2Efrac__add *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ej$

Definition 18 *We define* c_2Efrac_2Efrac__sub *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Definition 19 *We define* c_2Erat_2Erat__equiv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Let $c\_2Erat\_2Eabs\_\_rat\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Eabs\_\_rat\_\_CLASS \in (ty\_2Erat\_2Erat^{(2^{ty\_2Efrac\_2Efrac})}) \tag{18}$$

Definition 20 *We define* c_2Erat_2Eabs__rat *to be* $\lambda V0r \in ty\_2Efrac\_2Efrac.(ap\ c\_2Erat\_2Eabs\_\_rat\_\_C1$

Definition 21 *We define* c_2Erat_2Erat__sub *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 22 *We define* c_2Erat_2Erat__add *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 23 *We define* c_2Erat_2Erat__ainv *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.(ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ c\_2$

Definition 24 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t)).$

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Definition 25 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Definition 26 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E$\ V0t)\ $c_2Ebool_2E

Assume the following.
$$True \tag{19}$$

Assume the following.
$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{20}$$

Assume the following.
$$(\forall V0t \in 2.(((p\ V0t)\Rightarrow False)\Rightarrow(\neg(p\ V0t)))) \tag{21}$$

Assume the following.
$$(\forall V0t \in 2.((\neg(p\ V0t))\Rightarrow((p\ V0t)\Rightarrow False))) \tag{22}$$

Assume the following.
$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t))))))))) \tag{23}$$

Assume the following.
$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge((\neg False)\Leftrightarrow True))) \tag{24}$$

Assume the following.
$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{25}$$

Assume the following.
$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.(\\ (ap\ (ap\ \text{c\_2Erat\_2Erat\_\_add}\ V0a)\ V1b) = (ap\ (ap\ \text{c\_2Erat\_2Erat\_\_add}\\ V1b)\ V0a)))) \tag{26}$$

Assume the following.
$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.\\ ((ap\ (ap\ \text{c\_2Erat\_2Erat\_\_sub}\ V0r1)\ V1r2) = (ap\ (ap\ \text{c\_2Erat\_2Erat\_\_add}\\ V0r1)\ (ap\ \text{c\_2Erat\_2Erat\_\_ainv}\ V1r2))))) \tag{27}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.((ap \; \mathsf{c\_2Erat\_2Erat\_ainv} \; (ap \; \mathsf{c\_2Erat\_2Erat\_ainv} \\ V0r1)) = V0r1))$$

$$(28)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. \\ ((ap \; \mathsf{c\_2Erat\_2Erat\_ainv} \; (ap \; (ap \; \mathsf{c\_2Erat\_2Erat\_add} \; V0r1) \; V1r2)) = \\ (ap \; (ap \; \mathsf{c\_2Erat\_2Erat\_add} \; (ap \; \mathsf{c\_2Erat\_2Erat\_ainv} \; V0r1)) \; (ap \\ \mathsf{c\_2Erat\_2Erat\_ainv} \; V1r2)))))$$

$$(29)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p \; V0t))) \Leftrightarrow (p \; V0t))) \qquad (30)$$

Assume the following.

$$(\forall V0A \in 2.((p \; V0A) \Rightarrow ((\neg(p \; V0A)) \Rightarrow False))) \qquad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \; V0A) \lor (p \; V1B))) \Rightarrow False) \Leftrightarrow \\ (((p \; V0A) \Rightarrow False) \Rightarrow ((\neg(p \; V1B)) \Rightarrow False))))) \qquad (32)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \; V0A)) \lor (p \; V1B))) \Rightarrow False) \Leftrightarrow \\ ((p \; V0A) \Rightarrow ((\neg(p \; V1B)) \Rightarrow False))))) \qquad (33)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \; V0A)) \Rightarrow False) \Rightarrow (((p \; V0A) \Rightarrow False) \Rightarrow False))) \qquad (34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \; V0p) \Leftrightarrow (\\ (p \; V1q) \Leftrightarrow (p \; V2r))) \Leftrightarrow (((p \; V0p) \lor ((p \; V1q) \lor (p \; V2r))) \land (((p \; V0p) \lor ((\neg(\\ p \; V2r)) \lor (\neg(p \; V1q)))) \land (((p \; V1q) \lor ((\neg(p \; V2r)) \lor (\neg(p \; V0p)))) \land ((p \; V2r) \lor \\ ((\neg(p \; V1q)) \lor (\neg(p \; V0p))))))))))))$$

$$(35)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \; V0p) \Leftrightarrow (\\ (p \; V1q) \Rightarrow (p \; V2r))) \Leftrightarrow (((p \; V0p) \lor (p \; V1q)) \land (((p \; V0p) \lor (\neg(p \; V2r))) \land ((\\ \neg(p \; V1q)) \lor ((p \; V2r) \lor (\neg(p \; V0p)))))))))))$$

$$(36)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \; V0p) \Leftrightarrow (\neg(p \; V1q))) \Leftrightarrow (((p \; V0p) \lor \\ (p \; V1q)) \land ((\neg(p \; V1q)) \lor (\neg(p \; V0p)))))))$$

$$(37)$$

Theorem 1

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat. \\ ((ap \; \mathsf{c\_2Erat\_2Erat\_ainv} \; (ap \; (ap \; \mathsf{c\_2Erat\_2Erat\_sub} \; V0r1) \; V1r2)) = \\ (ap \; (ap \; \mathsf{c\_2Erat\_2Erat\_sub} \; V1r2) \; V0r1))))$$