

thm_2Erat_2ERAT__DIVDIV__MUL (TM- Luei3wMVgkEwcZHUgRhS6Qxre1mvZdNQv)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_{27a}}))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2. V0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t))$

Definition 7 We define `c_2Emarker_2EAC` to be $\lambda V0b1 \in 2. \lambda V1b2 \in 2. (\text{ap } (\text{ap } (\text{c_2Ebool_2E_2F_5C } V0b1) V1b2))$

Let $\text{ty_2Efrac_2Efrac} : \iota$ be given. Assume the following.

$$\text{nonempty ty_2Efrac_2Efrac} \tag{1}$$

Let $\text{ty_2Erat_2Erat} : \iota$ be given. Assume the following.

$$\text{nonempty ty_2Erat_2Erat} \tag{2}$$

Let $\text{c_2Erat_2Erep_rat_CLASS} : \iota$ be given. Assume the following.

$$\text{c_2Erat_2Erep_rat_CLASS} \in ((2^{\text{ty_2Efrac_2Efrac}})^{\text{ty_2Erat_2Erat}}) \tag{3}$$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define `c_2Erat_2Erep_rat` to be $\lambda V0a \in \text{ty_2Erat_2Erat}. (\text{ap } (\text{c_2Emin_2E_40 } \text{ty_2Efrac_2Efrac } V0a))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{13}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{14}$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \tag{15}$$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})(ty_2Epair_2Eprod\ ty_2Enum_2Enum)) \tag{16}$$

Definition 15 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. \lambda V1T2 \in ty_2Einteger.$

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 17 We define $c_2Einteger_2EABS$ to be $\lambda V0n \in ty_2Einteger_2Eint. (ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \tag{17}$$

Definition 18 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac. (ap\ (c_2Epair_2ESND\ t$

Definition 19 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{18}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{19}$$

Definition 20 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{20}$$

Definition 21 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic$

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum) \quad (25)$$

Definition 37 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.$

Definition 38 We define $c_2Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac.$

Definition 39 We define $c_2Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.$

Definition 40 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 41 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 42 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a}))\ A_27a))$

Let $c_2Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Earithmetic_2Enum_CASE\ A_27a \in (((A_27a^{(A_27a^{ty_2Enum_2Enum})})^{A_27a})^{ty_2Enum_2Enum}) \quad (26)$$

Definition 43 We define $c_2Efrac_2Efrac_0$ to be $(ap\ c_2Efrac_2Eabs_frac\ (ap\ (ap\ (c_2Epair_2E2C\ ty_2Enum_2Enum))))$

Definition 44 We define $c_2Erat_2Erat_0$ to be $(ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_0)$.

Definition 45 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$

Definition 46 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ A_27a\ V0P))))$

Definition 47 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ (c_2Ebool_2E21\ A_27a\ V0R))$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (27)$$

Definition 48 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).$

Definition 49 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27a.$

Definition 50 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).$

Definition 51 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).$

Definition 52 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).$

Definition 53 We define $c_2Erat_2Erat_of_num$ to be $(ap\ (ap\ (c_2Erelation_2EWFREC\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (32)$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (36)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))))) \quad (37)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in ((A_27a^{A_27a})^{A_27a}). \\
& \quad ((\forall V1x \in A_27a.(\forall V2y \in A_27a.(\forall V3z \in A_27a. \\
& ((ap\ (ap\ V0f\ V1x)\ (ap\ (ap\ V0f\ V2y)\ V3z)) = (ap\ (ap\ V0f\ (ap\ (ap\ V0f\ V1x) \\
& \quad V2y))\ V3z)))) \Rightarrow ((\forall V4x \in A_27a.(\forall V5y \in A_27a.((ap \\
& (ap\ V0f\ V4x)\ V5y) = (ap\ (ap\ V0f\ V5y)\ V4x)))) \Rightarrow (\forall V6x \in A_27a.(\\
& \quad \forall V7y \in A_27a.(\forall V8z \in A_27a.((ap\ (ap\ V0f\ V6x)\ (ap\ (ap \\
& V0f\ V7y)\ V8z)) = (ap\ (ap\ V0f\ V7y)\ (ap\ (ap\ V0f\ V6x)\ V8z))))))))) \\
& \hspace{15em} (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erat_2Erat.(\forall V1b \in ty_2Erat_2Erat.(\\
& \quad \forall V2c \in ty_2Erat_2Erat.((ap\ (ap\ c_2Erat_2Erat_mul\ V0a) \\
& (ap\ (ap\ c_2Erat_2Erat_mul\ V1b)\ V2c)) = (ap\ (ap\ c_2Erat_2Erat_mul \\
& \quad (ap\ (ap\ c_2Erat_2Erat_mul\ V0a)\ V1b))\ V2c)))))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erat_2Erat.(\forall V1b \in ty_2Erat_2Erat.(\\
& (ap\ (ap\ c_2Erat_2Erat_mul\ V0a)\ V1b) = (ap\ (ap\ c_2Erat_2Erat_mul \\
& \quad V1b)\ V0a)))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat.(\forall V1r2 \in ty_2Erat_2Erat. \\
& ((ap\ (ap\ c_2Erat_2Erat_div\ V0r1)\ V1r2) = (ap\ (ap\ c_2Erat_2Erat_mul \\
& \quad V0r1)\ (ap\ c_2Erat_2Erat_minv\ V1r2)))))) \\
& \hspace{15em} (41)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erat_2Erat.(\forall V1b \in ty_2Erat_2Erat.(\\
& ((\neg(V0a = (ap\ c_2Erat_2Erat_of_num\ c_2Enum_2E0))) \wedge (\neg(V1b = \\
& (ap\ c_2Erat_2Erat_of_num\ c_2Enum_2E0)))) \Rightarrow ((ap\ c_2Erat_2Erat_minv \\
& (ap\ (ap\ c_2Erat_2Erat_mul\ V0a)\ V1b)) = (ap\ (ap\ c_2Erat_2Erat_mul \\
& \quad (ap\ c_2Erat_2Erat_minv\ V0a))\ (ap\ c_2Erat_2Erat_minv\ V1b)))))) \\
& \hspace{15em} (42)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0b \in ty_2Erat_2Erat.(\forall V1d \in ty_2Erat_2Erat.(\\
& \quad \forall V2a \in ty_2Erat_2Erat.(\forall V3c \in ty_2Erat_2Erat.((\\
& (\neg(V0b = (ap\ c_2Erat_2Erat_of_num\ c_2Enum_2E0))) \wedge (\neg(V1d = (\\
& ap\ c_2Erat_2Erat_of_num\ c_2Enum_2E0)))) \Rightarrow ((ap\ (ap\ c_2Erat_2Erat_mul \\
& (ap\ (ap\ c_2Erat_2Erat_div\ V2a)\ V0b))\ (ap\ (ap\ c_2Erat_2Erat_div \\
& V3c)\ V1d)) = (ap\ (ap\ c_2Erat_2Erat_div\ (ap\ (ap\ c_2Erat_2Erat_mul \\
& V2a)\ V3c))\ (ap\ (ap\ c_2Erat_2Erat_mul\ V0b)\ V1d)))))))))
\end{aligned}$$