

thm_2Erat_2ERAT__DIV__MULMINV (TMFpT4oSYF3hLaN61pBn9ZDTEH6M6C1Mngd)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \tag{3}$$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \tag{4}$$

Let $c_2Epair_2EFSST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFSST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{5}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Efrac_2Efrac_nrm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2EFSST\ ty_2Efrac_2Efrac$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \tag{6}$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum\ ty_2Eenum_2Eenum)})^{ty_2Einteger_2Eint}) \tag{7}$$

Definition 5 We define $c_2Emin_2E.40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** *(the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).*

Definition 6 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E.40 (ty$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)) \quad (8)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (9)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)}} \quad (10)$$

Definition 7 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)$

Definition 8 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint_neg$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (12)$$

Definition 9 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint)^{ty_2Enum_2Enum} \quad (13)$$

Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (14)$$

Definition 10 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.$

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E.21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Emin_2E.3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E.2F.5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E.21 2) (\lambda V2t \in 2.V2t)))$

Definition 14 We define c_Ebool_ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 15 We define $c_Einteger_EABS$ to be $\lambda V0n \in ty_Einteger_Eint. (ap (ap (ap (c_Ebool_ECOND$

Let $c_Epair_ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_Epair_ESND \\ A_27a\ A_27b \in (A_27b^{(ty_Epair_Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (15)$$

Definition 16 We define $c_Efrac_Efrac_dnm$ to be $\lambda V0f \in ty_Efrac_Efrac. (ap (c_Epair_ESND\ t$

Definition 17 We define $c_Earithmetic_EZERO$ to be c_EEnum_E0 .

Let $c_EEnum_EREP_num : \iota$ be given. Assume the following.

$$c_EEnum_EREP_num \in (\omega^{ty_EEnum_EEnum}) \quad (16)$$

Let $c_EEnum_ESUC_REP : \iota$ be given. Assume the following.

$$c_EEnum_ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 18 We define c_EEnum_ESUC to be $\lambda V0m \in ty_EEnum_EEnum. (ap\ c_EEnum_EABS_num$

Let $c_Earithmetic_E_EB : \iota$ be given. Assume the following.

$$c_Earithmetic_E_EB \in ((ty_EEnum_EEnum)^{ty_EEnum_EEnum})^{ty_EEnum_EEnum} \quad (18)$$

Definition 19 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_EEnum_EEnum. (ap (ap\ c_Earithmetic$

Definition 20 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_EEnum_EEnum. V0x$.

Definition 21 We define $c_EintExtension_ESGN$ to be $\lambda V0x \in ty_Einteger_Eint. (ap (ap (ap (c_Ebool$

Definition 22 We define $c_Efrac_Efrac_sgn$ to be $\lambda V0f1 \in ty_Efrac_Efrac. (ap\ c_EintExtension_ESGN$

Let $c_Einteger_Eint_mul : \iota$ be given. Assume the following.

$$\begin{aligned} c_Einteger_Eint_mul \in (((ty_Epair_Eprod\ ty_EEnum_EEnum \\ ty_EEnum_EEnum)^{(ty_Epair_Eprod\ ty_EEnum_EEnum\ ty_EEnum_EEnum)})^{(ty_Epair_Eprod\ ty_EEnum_EEnum\ ty_EEnum_EEnum)}) \end{aligned} \quad (19)$$

Definition 23 We define $c_Einteger_Eint_mul$ to be $\lambda V0T1 \in ty_Einteger_Eint. \lambda V1T2 \in ty_Einteger$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_Epair_EABS_prod \\ A_27a\ A_27b \in ((ty_Epair_Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (20)$$

Definition 24 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)}) \quad (21)$$

Definition 25 We define $c_2Efrac_2Efrac_minv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap\ c_2Efrac_2Eabs_frac$ Let $ty_2Erat_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \quad (22)$$

Let $c_2Erat_2Erep_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (23)$$

Definition 26 We define $c_2Erat_2Erep_rat$ to be $\lambda V0a \in ty_2Erat_2Erat.(ap\ (c_2Emin_2E_40\ ty_2Efrac$

Definition 27 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat^{(2^{ty_2Efrac_2Efrac})}) \quad (24)$$

Definition 28 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap\ c_2Erat_2Eabs_rat_CLASS$

Definition 29 We define $c_2Erat_2Erat_minv$ to be $\lambda V0r1 \in ty_2Erat_2Erat.(ap\ c_2Erat_2Eabs_rat\ (ap\ c$

Definition 30 We define $c_2Efrac_2Efrac_mul$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 31 We define $c_2Erat_2Erat_mul$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 32 We define $c_2Efrac_2Efrac_div$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 33 We define $c_2Erat_2Erat_div$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\begin{aligned} & ((\forall V0x \in ty_2Efrac_2Efrac.(\forall V1y \in ty_2Efrac_2Efrac. \\ & ((ap\ c_2Erat_2Eabs_rat\ (ap\ (ap\ c_2Efrac_2Efrac_mul\ (ap\ c_2Erat_2Erep_rat \\ & (ap\ c_2Erat_2Eabs_rat\ V0x)))\ V1y))) = (ap\ c_2Erat_2Eabs_rat\ (\\ & ap\ (ap\ c_2Efrac_2Efrac_mul\ V0x\ V1y)))))) \wedge (\forall V2x \in ty_2Efrac_2Efrac. \\ & (\forall V3y \in ty_2Efrac_2Efrac.((ap\ c_2Erat_2Eabs_rat\ (ap\ (\\ & ap\ c_2Efrac_2Efrac_mul\ V2x)\ (ap\ c_2Erat_2Erep_rat\ (ap\ c_2Erat_2Eabs_rat \\ & V3y)))))) = (ap\ c_2Erat_2Eabs_rat\ (ap\ (ap\ c_2Efrac_2Efrac_mul \\ & V2x\ V3y)))))) \end{aligned} \quad (27)$$

Theorem 1

$$(\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\ ((ap (ap c_2Erat_2Erat_div V0r1) V1r2) = (ap (ap c_2Erat_2Erat_mul \\ V0r1) (ap c_2Erat_2Erat_minv V1r2))))))$$