

Definition 7 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21) 2) (\lambda V2t \in 2)))$.
Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (6)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (7)$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap\ P\ x))$ **then** *(the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).*

Definition 9 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty_2Einteger_2Eint)))$.

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Einteger_2Etint}) \quad (8)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (9)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (10)$$

Definition 10 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$.

Definition 11 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint_neg)$.

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (11)$$

Definition 12 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$.

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (13)$$

Definition 13 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 14 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t) c_Ebool_2E_21))$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (15)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (16)$$

Definition 15 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 17 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 18 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 19 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 20 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (17)$$

Definition 21 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 22 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 23 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Definition 24 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (21)$$

Definition 25 We define $c_2Earithmic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod ty_2Einteger_2Eint ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (22)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (23)$$

Definition 26 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND ty$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (24)$$

Definition 27 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2EFST ty$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (25)$$

Definition 28 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 29 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 30 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda$

Definition 31 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Definition 32 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic$

Definition 33 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (26)$$

Definition 34 We define $c_2Efrac_2Efrac_1$ to be $(ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C ty_2$

Let $ty_2Erat_2Erat : \iota$ be given. Assume the following.

$$nonempty ty_2Erat_2Erat \quad (27)$$

Let $c_2Erat_2Erep_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (28)$$

Definition 35 We define $c_Erat_Erep_rat$ to be $\lambda V0a \in ty_2Erat_2Erat.(ap (c_Emin_2E40 ty_2Efrac$

Definition 36 We define $c_Efrac_2Efrac_mul$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 37 We define $c_Einteger_2EABS$ to be $\lambda V0n \in ty_2Einteger_2Eint.(ap (ap (ap (c_Ebool_2E0$

Definition 38 We define $c_EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint.(ap (ap (ap (c_Ebool$

Definition 39 We define $c_Efrac_2Efrac_sgn$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_EintExtension_2ES$

Definition 40 We define $c_Efrac_2Efrac_minv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_2Efrac_2Eabs_f$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat^{(2^{ty_2Efrac_2Efrac})}) \quad (29)$$

Definition 41 We define $c_Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap c_2Erat_2Eabs_rat_CL$

Definition 42 We define $c_Efrac_2Efrac_div$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 43 We define $c_Erat_2Erat_div$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 44 We define $c_Erat_2Erat_mul$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 45 We define $c_Erat_2Erat_minv$ to be $\lambda V0r1 \in ty_2Erat_2Erat.(ap c_2Erat_2Eabs_rat (ap c$

Definition 46 We define $c_Efrac_2Efrac_ainv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_2Efrac_2Eabs_f$

Definition 47 We define $c_Erat_2Erat_ainv$ to be $\lambda V0r1 \in ty_2Erat_2Erat.(ap c_2Erat_2Eabs_rat (ap c$

Definition 48 We define $c_Erat_2Erat_1$ to be $(ap c_2Erat_2Eabs_rat c_2Efrac_2Efrac_1)$.

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum))^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (30)$$

Definition 49 We define $c_Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Eintege$

Definition 50 We define $c_Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 51 We define $c_Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 52 We define $c_Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 53 We define $c_Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27$

Definition 54 We define $c_Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a}) A$

Let $c_2Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Earithmetic_2Enum_CASE\ A_27a \in \left((A_27a^{(A_27a^{ty_2Enum_2Enum})})_{A_27a} \right)_{ty_2Enum_2Enum} \quad (31)$$

Definition 55 We define $c_2Efrac_2Efrac_0$ to be $(ap\ c_2Efrac_2Eabs_frac\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Enum_2Enum\ c_2Efrac_2Efrac_0))))$.

Definition 56 We define $c_2Erat_2Erat_0$ to be $(ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_0)$.

Definition 57 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ (c_2Ebool_2E_2I\ c_2Efrac_2Efrac_0))$.

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (32)$$

Definition 58 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).(ap\ (c_2Ebool_2EARB\ A_27a))$.

Definition 59 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27a.(ap\ (c_2Ebool_2EARB\ A_27a))$.

Definition 60 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).(ap\ (c_2Ebool_2EARB\ A_27a))$.

Definition 61 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).(ap\ (c_2Ebool_2EARB\ A_27a))$.

Definition 62 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).(ap\ (c_2Ebool_2EARB\ A_27a))$.

Definition 63 We define $c_2Erat_2Erat_of_num$ to be $(ap\ (ap\ (c_2Erelation_2EWFREC\ ty_2Enum_2Enum\ c_2Efrac_2Efrac_0)))$.

Assume the following.

$$True \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (36)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty_2Einteger_2Eint. (\forall V1b \in ty_2Einteger_2Eint. \\ & ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0))\ V1b)) \Rightarrow ((ap\ c_2Efrac_2Efrac_nmr\ (ap\ c_2Efrac_2Eabs_frac \\ & (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint) \\ & V0a)\ V1b))) = V0a)))) \end{aligned} \quad (40)$$

Assume the following.

$$(\forall V0a \in ty_2Efrac_2Efrac. ((ap\ (ap\ c_2Efrac_2Efrac_mul\ V0a)\ c_2Efrac_2Efrac_1) = V0a)) \quad (41)$$

Assume the following.

$$((ap\ c_2Efrac_2Efrac_minv\ c_2Efrac_2Efrac_1) = c_2Efrac_2Efrac_1) \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap\ c_2Einteger_2Eint_of_num\ V0m) = (ap\ c_2Einteger_2Eint_of_num \\ & V1n)) \Leftrightarrow (V0m = V1n)))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\ & ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num \\ & V0n))\ (ap\ c_2Einteger_2Eint_of_num\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ & V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_neg \\ & (ap\ c_2Einteger_2Eint_of_num\ V0n))\ (ap\ c_2Einteger_2Eint_neg \\ & (ap\ c_2Einteger_2Eint_of_num\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ & V1m)\ V0n))) \wedge (((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_neg \\ & (ap\ c_2Einteger_2Eint_of_num\ V0n))\ (ap\ c_2Einteger_2Eint_of_num \\ & V1m))) \Leftrightarrow ((\neg(V0n = c_2Enum_2E0)) \vee (\neg(V1m = c_2Enum_2E0)))) \wedge ((p \\ & (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num \\ & V0n))\ (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num \\ & V1m)))) \Leftrightarrow False)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c_2Earithmic_2EBIT1\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m)))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO)\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO) \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& V0n)\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m)))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$(p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ ty_2Efrac_2Efrac\ ty_2Erat_2Erat) \\
c_2Erat_2Erat_equiv)\ c_2Erat_2Eabs_rat)\ c_2Erat_2Erep_rat)) \tag{48}$$

Assume the following.

$$((ap\ c_2Erat_2Erat_of_num\ (ap\ c_2Earithmic_2ENUMERAL\ (ap\ \\
c_2Earithmic_2EBIT1\ c_2Earithmic_2EZERO))) = (ap\ c_2Erat_2Eabs_rat \\
c_2Efrac_2Efrac_1)) \tag{49}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty_2Efrac_2Efrac. (\forall V1y \in ty_2Efrac_2Efrac. \\
& ((ap\ c_2Erat_2Eabs_rat (ap (ap\ c_2Efrac_2Efrac_mul (ap\ c_2Erat_2Erep_rat \\
& (ap\ c_2Erat_2Eabs_rat\ V0x)))\ V1y))) = (ap\ c_2Erat_2Eabs_rat (\\
& ap (ap\ c_2Efrac_2Efrac_mul\ V0x)\ V1y)))))) \wedge (\forall V2x \in ty_2Efrac_2Efrac. \\
& (\forall V3y \in ty_2Efrac_2Efrac. ((ap\ c_2Erat_2Eabs_rat (ap (\\
& ap\ c_2Efrac_2Efrac_mul\ V2x)\ (ap\ c_2Erat_2Erep_rat (ap\ c_2Erat_2Eabs_rat \\
& V3y)))))) = (ap\ c_2Erat_2Eabs_rat (ap (ap\ c_2Efrac_2Efrac_mul \\
& V2x)\ V3y))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f1 \in ty_2Efrac_2Efrac. ((\neg((ap\ c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0) = (ap\ c_2Efrac_2Efrac_nmr\ V0f1))) \Rightarrow ((ap\ c_2Erat_2Erat_minv \\
& (ap\ c_2Erat_2Eabs_rat\ V0f1)) = (ap\ c_2Erat_2Eabs_rat (ap\ c_2Efrac_2Efrac_minv \\
& V0f1))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\neg((ap\ c_2Erat_2Erat_of_num (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) = (ap\ c_2Erat_2Erat_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& ((ap (ap\ c_2Erat_2Erat_div\ V0r1)\ V1r2) = (ap (ap\ c_2Erat_2Erat_mul \\
& V0r1)\ (ap\ c_2Erat_2Erat_minv\ V1r2))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. ((ap\ c_2Erat_2Erat_ainv (ap\ c_2Erat_2Erat_ainv \\
& V0r1)) = V0r1))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& ((ap\ c_2Erat_2Erat_ainv (ap (ap\ c_2Erat_2Erat_mul\ V0r1)\ V1r2)) = \\
& (ap (ap\ c_2Erat_2Erat_mul\ V0r1)\ (ap\ c_2Erat_2Erat_ainv\ V1r2))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& (((ap\ c_2Erat_2Erat_ainv\ V0r1) = V1r2) \Leftrightarrow (V0r1 = (ap\ c_2Erat_2Erat_ainv \\
& V1r2))))))
\end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat. ((\neg(V0r1 = (ap\ c_2Erat_2Erat_of_num\ c_2Enum_2E0))) \Rightarrow ((ap\ c_2Erat_2Erat_ainv\ (ap\ c_2Erat_2Erat_minv\ V0r1)) = (ap\ c_2Erat_2Erat_minv\ (ap\ c_2Erat_2Erat_ainv\ V0r1)))))) \quad (57)$$

Assume the following.

$$(c_2Erat_2Erat_1 = (ap\ c_2Erat_2Erat_of_num\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \quad (58)$$

Theorem 1

$$(\forall V0r \in ty_2Erat_2Erat. ((ap\ (ap\ c_2Erat_2Erat_div\ V0r)\ (ap\ c_2Erat_2Erat_ainv\ (ap\ c_2Erat_2Erat_of_num\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) = (ap\ c_2Erat_2Erat_ainv\ V0r)))$$