

thm_2Erat_2ERAT__EQ0__NMR
(TMFNfU8n4gTCa1ihotTTesjAv7NzXmo1Rdi)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow Q)$ of type ι .

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty ty_2Efrac_2Efrac \tag{3}$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty ty_2Einteger_2Eint \tag{4}$$

Let $c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod ty_2Einteger_2Eint ty_2Einteger_2Eint)}) \tag{5}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{6}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{7}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{8}$$

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Definition 9 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{9}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{10}$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$.

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda P : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ P))))$.

Definition 14 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint_REP_CLASS}) \tag{11}$$

Definition 15 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E_40\ a)\ V0a)$.

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \tag{12}$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \tag{13}$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \tag{14}$$

Definition 16 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 17 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (15)$$

Definition 18 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B))$

Definition 19 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B))$

Definition 20 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (16)$$

Definition 21 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (17)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2ESND\ A.27a\ A.27b \in (A.27b^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}) \quad (18)$$

Definition 22 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2ESND\ t))$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (19)$$

Definition 23 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Let $ty_2Erat_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \quad (20)$$

Let $c_2Erat_2Erep_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (21)$$

Definition 24 We define $c_2Erat_2Erep_rat$ to be $\lambda V0a \in ty_2Erat_2Erat.(ap\ (c_2Emin_2E40\ ty_2Efrac_2Efrac))$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (22)$$

Definition 25 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2EFST\ ty_2Efrac_2Efrac_nmr\ f))$

Definition 26 We define $c_2Erat_2Erat_nmr$ to be $\lambda V0r \in ty_2Erat_2Erat.(ap\ c_2Efrac_2Efrac_nmr\ (ap\ c_2Efrac_2Efrac_nmr\ r))$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (23)$$

Definition 27 We define $c_2Efrac_2Efrac_0$ to be $(ap\ c_2Efrac_2Efrac_0\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Efrac_2Efrac_0\ f))\ f))$

Definition 28 We define $c_2Efrac_2Efrac_1$ to be $(ap\ c_2Efrac_2Efrac_1\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Efrac_2Efrac_1\ f))\ f))$

Definition 29 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac.(ap\ c_2Erat_2Erat_equiv\ f1\ f2)$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat^{(2^{ty_2Efrac_2Efrac})}) \quad (24)$$

Definition 30 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap\ c_2Erat_2Eabs_rat_CLASS\ r)$

Definition 31 We define $c_2Erat_2Erat_1$ to be $(ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_1)$.

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (25)$$

Definition 32 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint_add\ T1\ T2)$

Definition 33 We define $c_2Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac.(ap\ c_2Efrac_2Efrac_add\ f1\ f2)$

Definition 34 We define $c_2Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap\ c_2Erat_2Erat_add\ r1\ r2)$

Definition 35 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 36 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 37 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a}))\ A_27a))$

Let $c_2Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Earithmetic_2Enum_CASE\ A_27a \in \\ (((A_27a^{(A_27a^{ty_2Enum_2Enum})})^{A_27a})^{ty_2Enum_2Enum}) \end{aligned} \quad (26)$$

Definition 38 We define $c_Erat_Erat_0$ to be $(ap\ c_Erat_Eabs_rat\ c_Efrac_Efrac_0)$.

Definition 39 We define $c_ERelation_EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ (c_Ebool_E2E_21$
Let $c_Ebool_EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ebool_EARB\ A_27a \in A_27a \quad (27)$$

Definition 40 We define c_Ebool_ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 41 We define $c_ERelation_ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1f$

Definition 42 We define $c_ERelation_ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b$

Definition 43 We define $c_ERelation_Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 44 We define $c_ERelation_Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 45 We define $c_ERelation_EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 46 We define $c_Erat_Erat_of_num$ to be $(ap\ (ap\ (c_ERelation_EWFREC\ ty_Eenum_Eenum$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty_Einteger_Eint.(\forall V1b \in ty_Einteger_Eint. \\ & ((p\ (ap\ (ap\ c_Einteger_Eint_lt\ (ap\ c_Einteger_Eint_of_num \\ & c_Eenum_E0))\ V1b)) \Rightarrow ((ap\ c_Efrac_Efrac_nmr\ (ap\ c_Efrac_Eabs_frac \\ & (ap\ (ap\ (c_Epair_E2C\ ty_Einteger_Eint\ ty_Einteger_Eint) \\ & V0a)\ V1b))) = V0a)))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty_Einteger_Eint.(\forall V1b \in ty_Einteger_Eint. \\ & ((p\ (ap\ (ap\ c_Einteger_Eint_lt\ (ap\ c_Einteger_Eint_of_num \\ & c_Eenum_E0))\ V1b)) \Rightarrow ((ap\ c_Efrac_Efrac_dnm\ (ap\ c_Efrac_Eabs_frac \\ & (ap\ (ap\ (c_Epair_E2C\ ty_Einteger_Eint\ ty_Einteger_Eint) \\ & V0a)\ V1b))) = V1b)))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
 & V0x) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
 & (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) = V0x))
 \end{aligned}
 \tag{33}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
 & (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0x) = (ap c_2Einteger_2Eint_of_num \\
 & c_2Enum_2E0)))
 \end{aligned}
 \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& \quad ((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_of_num \\
& \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad (ap c_2Arithmetic_2EBIT1 V0n)))))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt \\
& (ap c_2Integer_2Eint_of_num c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num \\
& (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT2 V0n)))))) \Leftrightarrow \\
& \quad True) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_of_num \\
& \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow \\
& \quad False) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_of_num \\
& \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& (ap c_2Arithmetic_2ENUMERAL V0n)))))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Integer_2Eint_lt \\
& (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& V0n)) (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow False) \wedge \\
& \quad (((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_neg \\
& (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& (ap c_2Arithmetic_2EBIT1 V0n)))))) (ap c_2Integer_2Eint_of_num \\
& \quad c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap \\
& c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& (ap c_2Arithmetic_2EBIT2 V0n)))))) (ap c_2Integer_2Eint_of_num \\
& \quad c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap \\
& \quad c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL V0n))) \\
& \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad V1m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0n V1m))) \wedge (((p (ap (ap \\
& c_2Integer_2Eint_lt (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT1 V0n)))))) \\
& (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& V1m)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_neg \\
& (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& (ap c_2Arithmetic_2EBIT2 V0n)))))) (ap c_2Integer_2Eint_of_num \\
& (ap c_2Arithmetic_2ENUMERAL V1m)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt \\
& (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& V0n)) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& (ap c_2Arithmetic_2ENUMERAL V1m)))))) \Leftrightarrow False) \wedge ((p (ap (ap c_2Integer_2Eint_lt \\
& (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num (\\
& \quad ap c_2Arithmetic_2ENUMERAL V0n)))) (ap c_2Integer_2Eint_neg \\
& \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad V1m)))))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V1m V0n))))))))))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((ap c_2Erat_2Erat_of_num c_2Enum_2E0) = (ap c_2Erat_2Eabs_rat \\
& \quad c_2Efrac_2Efrac_0)) \hspace{10em} (36)
\end{aligned}$$

Assume the following.

$$(\forall V0r \in ty_2Erat_2Erat.((ap\ c_2Erat_2Eabs_rat\ (ap\ c_2Erat_2Erep_rat\ V0r)) = V0r)) \quad (37)$$

Assume the following.

$$(\forall V0f1 \in ty_2Efrac_2Efrac.(\forall V1f2 \in ty_2Efrac_2Efrac.(((ap\ c_2Erat_2Eabs_rat\ V0f1) = (ap\ c_2Erat_2Eabs_rat\ V1f2)) \Leftrightarrow (p\ (ap\ (ap\ c_2Erat_2Erat_equiv\ V0f1\ V1f2)))))) \quad (38)$$

Assume the following.

$$(\forall V0f1 \in ty_2Efrac_2Efrac.(((ap\ c_2Efrac_2Efrac_nmr\ (ap\ c_2Erat_2Erep_rat\ (ap\ c_2Erat_2Eabs_rat\ V0f1))) = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) \Leftrightarrow ((ap\ c_2Efrac_2Efrac_nmr\ V0f1) = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)))) \quad (39)$$

Theorem 1

$$(\forall V0r1 \in ty_2Erat_2Erat.((V0r1 = (ap\ c_2Erat_2Erat_of_num\ c_2Enum_2E0)) \Leftrightarrow ((ap\ c_2Erat_2Erat_nmr\ V0r1) = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))))$$