

thm_2Erat_2ERAT__EQUIV__NMR__LTZ__IFF
(TMQcY7Y8eC31mbRgrmrcAjd2cYMbVZ8B39k)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 0t \in 2.V 0t))$.

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \tag{1}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A 0 A 1) \tag{2}$$

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Einteger_2Eint} \tag{3}$$

Let `c_2Einteger_2Eint__REP__CLASS` : ι be given. Assume the following.

$$\text{c_2Einteger_2Eint_REP_CLASS} \in ((2^{(\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum } \text{ty_2Enum_2Enum})}) \text{ty_2Einteger_2Eint}) \tag{4}$$

Definition 5 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 6 We define `c_2Einteger_2Eint__REP` to be $\lambda V 0a \in \text{ty_2Einteger_2Eint}. (\text{ap } (\text{c_2Emin_2E_40 } (\text{ty_2Einteger_2Eint_REP_CLASS } a)))$

Let `c_2Einteger_2Etint__lt` : ι be given. Assume the following.

$$\text{c_2Einteger_2Etint_lt} \in ((2^{(\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum } \text{ty_2Enum_2Enum})}) (\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum})) \tag{5}$$

Definition 7 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 8 We define $c_2Einteger_2Eint_gt$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Definition 9 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow Q)$ of type ι .

Definition 10 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E7E))$

Definition 11 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Definition 12 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21 2) (\lambda V2t \in 2)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{6}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{7}$$

Definition 13 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \tag{8}$$

Definition 14 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21 2) (\lambda V2t \in 2)))$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \tag{9}$$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \tag{10}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{11}$$

Definition 15 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND\ f))$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{12}$$

Definition 16 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2EFST ty_2Enum_2Enum))$. Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum))^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (13)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}))^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (14)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})} \quad (15)$$

Definition 17 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$.

Definition 18 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$.

Definition 19 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$.

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in ty_2Efrac_2Efrac.(p (ap (ap c_2Einteger_2Eint_lt \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap c_2Efrac_2Efrac_dnm \\
& V0f)))) \\
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. \\
& ((\neg(p (ap (ap c_2Einteger_2Eint_lt V0x) V1y))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le \\
& V1y) V0x)))))) \\
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. \\
& ((p (ap (ap c_2Einteger_2Eint_lt V0x) V1y)) \Rightarrow (p (ap (ap c_2Einteger_2Eint_le \\
& V0x) V1y)))))) \\
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint.(\forall V1q \in ty_2Einteger_2Eint. \\
& (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_mul V0p) V1q))) \Leftrightarrow (((\\
& p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V0p)) \wedge (p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V1q))) \vee ((p (ap (ap c_2Einteger_2Eint_lt V0p) (\\
& ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \wedge (p (ap (ap c_2Einteger_2Eint_lt \\
& V1q) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)))))) \wedge ((p (\\
& ap (ap c_2Einteger_2Eint_lt (ap (ap c_2Einteger_2Eint_mul V0p) \\
& V1q)) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow (((p (ap \\
& (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \\
& V0p)) \wedge (p (ap (ap c_2Einteger_2Eint_lt V1q) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)))))) \vee ((p (ap (ap c_2Einteger_2Eint_lt V0p) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))) \wedge (p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V1q))))))))) \\
\end{aligned} \tag{25}$$

Theorem 1

$$\begin{aligned}
& (\forall V0a \in ty_2Efrac_2Efrac.(\forall V1b \in ty_2Efrac_2Efrac. \\
& ((p (ap (ap c_2Erat_2Erat_equiv V0a) V1b)) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_lt \\
& (ap c_2Efrac_2Efrac_nmr V0a)) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt (ap c_2Efrac_2Efrac_nmr \\
& V1b)) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))))))))) \\
\end{aligned}$$