

# thm\_2Erat\_2ERAT\_\_EQUIV\_\_TRANS (TMJYfp4ps6Bi6VY15kq9zs56hv6YYL2c4HG)

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**Definition 1** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

**Definition 5** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

**Definition 6** We define `c_2Ebool_2EF` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let `ty_2Einteger_2Eint` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{3}$$

Let `c_2Einteger_2Eint__REP__CLASS` :  $\iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})ty\_2Einteger\_2Eint) \tag{4}$$

**Definition 8** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E\_40 (ty\_2Einteger\_2Eint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (5)$$

**Definition 10** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint$ .

**Definition 11** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E$  Let  $ty\_2Efrac\_2Efrac : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \quad (6)$$

Let  $c\_2Efrac\_2Erep\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Erep\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \quad (7)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (8)$$

**Definition 12** We define  $c\_2Efrac\_2Efrac\_dnm$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap (c\_2Epair\_2ESND\ t$  Let  $c\_2Einteger\_2Eint\_mul : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (9)$$

Let  $c\_2Einteger\_2Eint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (10)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \quad (11)$$

**Definition 13** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$ .

**Definition 14** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint$ .

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (12)$$

**Definition 15** We define  $c\_2Efrac\_2Efrac\_nmr$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap (c\_2Epair\_2EFST ty$

**Definition 16** We define  $c\_2Erat\_2Erat\_equiv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (13)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (14)$$

**Definition 17** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (17)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in ty\_2Efrac\_2Efrac.(p (ap (ap c\_2Einteger\_2Eint\_lt \\ & (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) (ap c\_2Efrac\_2Efrac\_dnm \\ & V0f)))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint. \\ & (\forall V2n \in ty\_2Einteger\_2Eint.((p (ap (ap c\_2Einteger\_2Eint\_lt \\ & (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) V2n)) \Rightarrow ((V0a = V1b) \Leftrightarrow \\ & ((ap (ap c\_2Einteger\_2Eint\_mul V0a) V2n) = (ap (ap c\_2Einteger\_2Eint\_mul \\ & V1b) V2n)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty\_2Einteger\_2Eint. (\forall V1x \in ty\_2Einteger\_2Eint. \\
& ((ap (ap c\_2Einteger\_2Eint\_mul V1x) V0y) = (ap (ap c\_2Einteger\_2Eint\_mul \\
& V0y) V1x))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\
& (\forall V2x \in ty\_2Einteger\_2Eint. ((ap (ap c\_2Einteger\_2Eint\_mul \\
& V2x) (ap (ap c\_2Einteger\_2Eint\_mul V1y) V0z)) = (ap (ap c\_2Einteger\_2Eint\_mul \\
& (ap (ap c\_2Einteger\_2Eint\_mul V2x) V1y)) V0z))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\
& (\forall V2z \in ty\_2Einteger\_2Eint. ((\neg(V0x = (ap c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0))) \Rightarrow ((V1y = V2z) \Leftrightarrow ((ap (ap c\_2Einteger\_2Eint\_mul \\
& V0x) V1y) = (ap (ap c\_2Einteger\_2Eint\_mul V0x) V2z))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Efrac\_2Efrac. (\forall V1b \in ty\_2Efrac\_2Efrac. \\
& ((p (ap (ap c\_2Erat\_2Erat\_equiv V0a) V1b)) \Rightarrow (((ap c\_2Efrac\_2Efrac\_nmr \\
& V0a) = (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) \Leftrightarrow ((ap c\_2Efrac\_2Efrac\_nmr \\
& V1b) = (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Efrac\_2Efrac. (\forall V1b \in ty\_2Efrac\_2Efrac. \\
& (((ap c\_2Efrac\_2Efrac\_nmr V0a) = (ap c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0)) \Rightarrow ((p (ap (ap c\_2Erat\_2Erat\_equiv V0a) V1b)) \Leftrightarrow (( \\
& ap c\_2Efrac\_2Efrac\_nmr V1b) = (ap c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0))))))
\end{aligned} \tag{26}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0a \in ty\_2Efrac\_2Efrac. (\forall V1b \in ty\_2Efrac\_2Efrac. \\
& (\forall V2c \in ty\_2Efrac\_2Efrac. (((p (ap (ap c\_2Erat\_2Erat\_equiv \\
& V0a) V1b)) \wedge (p (ap (ap c\_2Erat\_2Erat\_equiv V1b) V2c))) \Rightarrow (p (ap ( \\
& ap c\_2Erat\_2Erat\_equiv V0a) V2c))))))
\end{aligned}$$