

thm_2Erat_2ERAT__LES__AINV
(TMaKm4hAzCdRk81LvH3A93tE84YUGpet542)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 8 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$
Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 11 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B$

Definition 12 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (7)$$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (8)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \quad (11)$$

Let $c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)}) \quad (12)$$

Definition 14 We define $c_2Efrac_2Efrac_1$ to be $(ap\ c_2Efrac_2Eabs_frac\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2E$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (13)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (14)$$

Definition 15 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND t$
Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2EFST \\ A.27a A.27b \in (A.27a^{(ty_2Epair_2Eprod A.27a A.27b)}) \end{aligned} \quad (15)$$

Definition 16 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2EFST ty$
Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Einteger_2Eint_REP_CLASS}) \quad (16)$$

Definition 17 We define $c_2Emin_2E.40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$
of type $\iota \Rightarrow \iota$.

Definition 18 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E.40 t$
Let $c_2Einteger_2Eint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)^{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum})^{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}) \quad (18)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})} \quad (19)$$

Definition 19 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)$

Definition 20 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 21 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Let $ty_2Erat_2Erat : \iota$ be given. Assume the following.

$$nonempty ty_2Erat_2Erat \quad (20)$$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat)^{(2^{ty_2Efrac_2Efrac})} \quad (21)$$

Definition 22 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap c_2Erat_2Eabs_rat_CLASS r)$

Definition 23 We define $c_2Erat_2Erat_1$ to be $(ap c_2Erat_2Eabs_rat c_2Efrac_2Efrac_1)$.

Definition 39 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 40 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 41 We define $c_2Erat_2Erat_of_num$ to be $(ap (ap (c_2Erelation_2EWFREC ty_2Enum_2Enum$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod ty_2Enum_2Enum \\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \end{aligned} \quad (26)$$

Definition 42 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. (ap c_2Einteger_2Eint$

Definition 43 We define $c_2Efrac_2Efrac_ainv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. (ap c_2Efrac_2Eabs_fn$

Definition 44 We define $c_2Erat_2Erat_ainv$ to be $\lambda V0r1 \in ty_2Erat_2Erat. (ap c_2Erat_2Eabs_rat (ap c$

Definition 45 We define $c_2Efrac_2Efrac_sub$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. \lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 46 We define $c_2Erat_2Erat_sub$ to be $\lambda V0r1 \in ty_2Erat_2Erat. \lambda V1r2 \in ty_2Erat_2Erat. (ap$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (27)$$

Definition 47 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. \lambda V1T2 \in ty_2Einteger$

Definition 48 We define $c_2EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint. (ap (ap (ap (c_2Eboo$

Definition 49 We define $c_2Efrac_2Efrac_sgn$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. (ap c_2EintExtension_2ES$

Definition 50 We define $c_2Erat_2Erat_sgn$ to be $\lambda V0r \in ty_2Erat_2Erat. (ap c_2Efrac_2Efrac_sgn (ap c$

Definition 51 We define $c_2Erat_2Erat_les$ to be $\lambda V0r1 \in ty_2Erat_2Erat. \lambda V1r2 \in ty_2Erat_2Erat. (ap$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (31)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat. (\forall V1b \in ty_2Erat_2Erat. (\forall V2c \in ty_2Erat_2Erat. ((ap (ap c_2Erat_2Erat_add V0a) (ap (ap c_2Erat_2Erat_add V1b) V2c)) = (ap (ap c_2Erat_2Erat_add (ap (ap c_2Erat_2Erat_add V0a) V1b)) V2c)))))) \quad (32)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat. (\forall V1b \in ty_2Erat_2Erat. (ap (ap c_2Erat_2Erat_add V0a) V1b) = (ap (ap c_2Erat_2Erat_add V1b) V0a)))) \quad (33)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat. ((ap (ap c_2Erat_2Erat_add (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) V0a) = V0a)) \quad (34)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat. ((ap (ap c_2Erat_2Erat_add (ap c_2Erat_2Erat_ainv V0a)) V0a) = (ap c_2Erat_2Erat_of_num c_2Enum_2E0)))) \quad (35)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. (\forall V2r3 \in ty_2Erat_2Erat. ((p (ap (ap c_2Erat_2Erat_les (ap (ap c_2Erat_2Erat_add V0r1) V2r3)) (ap (ap c_2Erat_2Erat_add V1r2) V2r3))) \Leftrightarrow (p (ap (ap c_2Erat_2Erat_les V0r1) V1r2)))))) \quad (36)$$

Theorem 1

$$(\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. ((p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_ainv V0r1)) (ap c_2Erat_2Erat_ainv V1r2))) \Leftrightarrow (p (ap (ap c_2Erat_2Erat_les V1r2) V0r1))))))$$