

thm_2Erat_2ERAT_LSUB_EQ
 (TMMPE9oP7sud2Jp1aAaZW1fG3bc7D6jYBb5)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap \ (ap \ (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 4 We define c_2Enum_2E0 to be $(ap \ c_2Enum_2EABS_num \ c_2Enum_2EZERO_REP)$.

Definition 5 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREPR_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREPR_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap \ (ap \ (c_2Emin_2E_3D \ (2^{A_27a})) \ (\lambda V1x \in 2.V1x)) \ (\lambda V2y \in 2.V2y)))$

Definition 7 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap \ c_2Enum_2EABS_num \ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 8 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2B n))$

Definition 9 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (7)$$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (8)$$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod \\ A0\ A1) \end{aligned} \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (10)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2Eprod\ A_27a\ A_27b))$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \quad (11)$$

Let $c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)}) \quad (12)$$

Definition 12 We define $c_2Efrac_2Efrac_1$ to be $(ap\ c_2Efrac_2Eabs_frac\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Efrac_2Efrac)))$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (13)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (14)$$

Definition 13 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND t$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epair_2EFST \\ & A_27a \ A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a \ A_27b)}) \end{aligned} \quad (15)$$

Definition 14 We define $c_2Efrac_2Efrac_nrm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2EFST t$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (16)$$

Definition 15 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 16 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (t$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (17)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (18)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}} \quad (19)$$

Definition 17 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum).$

Definition 18 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.$

Definition 19 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac.$

Let $ty_2Erat_2Erat : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Erat_2Erat \quad (20)$$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat)^{(2^{ty_2Efrac_2Efrac})}) \quad (21)$$

Definition 20 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap c_2Erat_2Eabs_rat_CLASS t$

Definition 21 We define $c_2Erat_2Erat_1$ to be $(ap c_2Erat_2Eabs_rat c_2Efrac_2Efrac_1).$

Let $c_2Erat_2Erep_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (22)$$

Definition 22 We define $c_2Erat_2Erep_rat$ to be $\lambda V0a \in ty_2Erat_2Erat.(ap(c_2Emin_2E_40\ ty_2Efrac_2Efrac\ (ty_2Efrac_2Efrac\ A_27a\ B_27a)))$

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum})^{ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum})^{ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum}) \quad (23)$$

Definition 23 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.(ap(c_2Emin_2E_40\ ty_2Eint_add\ (ty_2Eint_add\ A_27a\ B_27a)))$

Definition 24 We define $c_2Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac.(ap(c_2Emin_2E_40\ ty_2Efrac_add\ (ty_2Efrac_add\ A_27a\ B_27a)))$

Definition 25 We define $c_2Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap(c_2Emin_2E_40\ ty_2Erat_add\ (ty_2Erat_add\ A_27a\ B_27a)))$

Definition 26 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 27 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})^{A_27a}))$

Definition 28 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap(ap(c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A_27a)))$

Let $c_2Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty}\ A_27a \Rightarrow c_2Earithmetic_2Enum_CASE\ A_27a \in (((A_27a^{(A_27a^{ty_2Enum_2Enum})})^{A_27a})^{ty_2Enum_2Enum}) \quad (24)$$

Definition 29 We define $c_2Efrac_2Efrac_0$ to be $(ap\ c_2Efrac_2Eabs_frac\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Efrac_2Efrac_0))))$

Definition 30 We define $c_2Erat_2Erat_0$ to be $(ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_0)$.

Definition 31 We define c_2Ebool_2EF to be $(ap(c_2Ebool_2E_21\ 2))\ (\lambda V0t \in 2.V0t))$.

Definition 32 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap(ap(c_2Emin_2E_3D_3D_3E\ V0t)))\ c_2Ebool_2E_7E)$

Definition 33 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (c_2Emin_2E_40\ V0P)))$

Definition 34 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap(c_2Ebool_2E_21\ (\lambda V0R)))$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty}\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (25)$$

Definition 35 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.((2^{V1t1})^{V2t2})))$

Definition 36 We define $c_2Erelation_2ERESTRUCT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1f \in ((A_27b^{A_27a})^{A_27a}))$

Definition 37 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27b.((\lambda V1b \in A_27b.((2^{V1a})^{V1b})^{V2b})))$

Definition 38 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 39 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 40 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 41 We define $c_2Erat_2Erat_of_num$ to be $(ap (ap (c_2Erelation_2EWFREC ty_2Enum_2Enum) ty_2Enum_2Enum))$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Einteger_2Etint_neg &\in ((ty_2Epair_2Eprod ty_2Enum_2Enum \\ &ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \end{aligned} \quad (26)$$

Definition 42 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. (ap c_2Einteger_2Eint$

Definition 43 We define $c_2Efrac_2Efrac_ainv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. (ap c_2Efrac_2Eabs_frac$

Definition 44 We define $c_2Erat_2Erat_ainv$ to be $\lambda V0r1 \in ty_2Erat_2Erat. (ap c_2Erat_2Eabs_rat (ap c_2Erat_2Erat_of_num$

Definition 45 We define $c_2Efrac_2Efrac_sub$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. \lambda V1f2 \in ty_2Efrac_2Efrac. (ap c_2Efrac_2Efrac_sub$

Definition 46 We define $c_2Erat_2Erat_sub$ to be $\lambda V0r1 \in ty_2Erat_2Erat. \lambda V1r2 \in ty_2Erat_2Erat. (ap c_2Erat_2Erat_sub$

Assume the following.

$$True \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (28)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (30)$$

Assume the following.

$$\begin{aligned} &(\forall V0a \in ty_2Erat_2Erat. (\forall V1b \in ty_2Erat_2Erat. (\forall V2c \in ty_2Erat_2Erat. ((ap (ap c_2Erat_2Erat_add V0a) \\ &(ap (ap c_2Erat_2Erat_add V1b) V2c)) = (ap (ap c_2Erat_2Erat_add \\ &(ap (ap c_2Erat_2Erat_add V0a) V1b)) V2c))))))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat. (\forall V1b \in ty_2Erat_2Erat. (ap (ap c_2Erat_2Erat_add V0a) V1b) = (ap (ap c_2Erat_2Erat_add V1b) V0a)))) \quad (32)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat. ((ap (ap c_2Erat_2Erat_add V0a) (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) = V0a)) \quad (33)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat. ((ap (ap c_2Erat_2Erat_add (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) V0a) = V0a)) \quad (34)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat. ((ap (ap c_2Erat_2Erat_add (ap c_2Erat_2Erat_ainv V0a)) V0a) = (ap c_2Erat_2Erat_of_num c_2Enum_2E0))) \quad (35)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. ((ap (ap c_2Erat_2Erat_sub V0r1) V1r2) = (ap (ap c_2Erat_2Erat_add V0r1) (ap c_2Erat_2Erat_ainv V1r2)))))) \quad (36)$$

Theorem 1

$$(\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. (\forall V2r3 \in ty_2Erat_2Erat. (((ap (ap c_2Erat_2Erat_sub V0r1) V1r2) = V2r3) \Leftrightarrow (V0r1 = (ap (ap c_2Erat_2Erat_add V1r2) V2r3)))))))$$