

thm_2Erat_2ERAT__LSUB__LES
(TMTzXU947EucxRszTNtSDdF6prsEBkWwwPN)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ c_2Enum_2EREP_num\ c_2Enum_2ESUC_REP))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define `c_2Earithmic_2EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2EBIT1))$

Definition 8 We define `c_2Earithmic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (7)$$

Let `c_2Einteger_2Eint_of_num` : ι be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (8)$$

Let `ty_2Efrac_2Efrac` : ι be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \quad (9)$$

Let `ty_2Erat_2Erat` : ι be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \quad (10)$$

Let `c_2Erat_2Erep_rat_CLASS` : ι be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (11)$$

Definition 9 We define `c_2Emin_2E40` to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 10 We define `c_2Erat_2Erep_rat` to be $\lambda V0a \in ty_2Erat_2Erat.(ap (c_2Emin_2E40 ty_2Efrac_2Efrac))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (12)$$

Let `c_2Efrac_2Erep_frac` : ι be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (13)$$

Let `c_2Epair_2EFST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2EFST\ A.27a\ A.27b \in (A.27a^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}) \quad (14)$$

Definition 11 We define `c_2Efrac_2Efrac_nmr` to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2EFST ty_2Efrac_2Efrac))$

Let `c_2Einteger_2Eint_REP_CLASS` : ι be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (15)$$

Definition 12 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (t$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod ty_2Enum_2Enum \\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \end{aligned} \quad (16)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (17)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})} \quad (18)$$

Definition 13 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$

Definition 14 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (19)$$

Definition 15 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 16 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 17 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow Q)$ of type ι .

Definition 18 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V0t))))$

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(A_27a t1 t2))))$

Definition 20 We define $c_2EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint.(ap (ap (ap (c_2Ebool_2E_21 2) (c_2Emin_2E_3D_3D_3E) (c_2Ebool_2E_2F_5C))))$

Definition 21 We define $c_2Efrac_2Efrac_sgn$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_2EintExtension_2ESGN (ap (c_2Ebool_2E_21 2) (c_2Emin_2E_3D_3D_3E) (c_2Ebool_2E_2F_5C))))$

Definition 22 We define $c_2Erat_2Erat_sgn$ to be $\lambda V0r \in ty_2Erat_2Erat.(ap c_2Efrac_2Efrac_sgn (ap (c_2Ebool_2E_21 2) (c_2Emin_2E_3D_3D_3E) (c_2Ebool_2E_2F_5C))))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \end{aligned} \quad (20)$$

Definition 23 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND (ap (c_2Ebool_2E_21 2) (c_2Emin_2E_3D_3D_3E) (c_2Ebool_2E_2F_5C))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (21)$$

Definition 24 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2E$

Let $c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)}) \quad (22)$$

Definition 25 We define $c_2Efrac_2Efrac_ainv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. (ap\ c_2Efrac_2Eabs_frac$

Let $c_2Einteger_2Eint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (23)$$

Definition 26 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. \lambda V1T2 \in ty_2Einteger$

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (24)$$

Definition 27 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. \lambda V1T2 \in ty_2Einteger$

Definition 28 We define $c_2Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. \lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 29 We define $c_2Efrac_2Efrac_sub$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. \lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 30 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. \lambda V1f2 \in ty_2Efrac_2Efrac$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat^{(2^{ty_2Efrac_2Efrac})}) \quad (25)$$

Definition 31 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac. (ap\ c_2Erat_2Eabs_rat_CLASS$

Definition 32 We define $c_2Erat_2Erat_sub$ to be $\lambda V0r1 \in ty_2Erat_2Erat. \lambda V1r2 \in ty_2Erat_2Erat. (ap$

Definition 33 We define $c_2Erat_2Erat_les$ to be $\lambda V0r1 \in ty_2Erat_2Erat. \lambda V1r2 \in ty_2Erat_2Erat. (ap$

Definition 34 We define $c_2Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat. \lambda V1r2 \in ty_2Erat_2Erat. (ap$

Definition 35 We define $c_2Erat_2Erat_ainv$ to be $\lambda V0r1 \in ty_2Erat_2Erat. (ap\ c_2Erat_2Eabs_rat\ (ap\ c_2E$

Definition 36 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 37 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Assume the following.

$$True \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge (((\neg(p\ V0A)) \vee (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \quad (40)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat.(\forall V1b \in ty_2Erat_2Erat.(\forall V2c \in ty_2Erat_2Erat.((ap\ (ap\ c.2Erat_2Erat_add\ V0a)\ (ap\ (ap\ c.2Erat_2Erat_add\ V1b)\ V2c)) = (ap\ (ap\ c.2Erat_2Erat_add\ (ap\ (ap\ c.2Erat_2Erat_add\ V0a)\ V1b))\ V2c)))))) \quad (41)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat.(\forall V1b \in ty_2Erat_2Erat.((ap\ (ap\ c.2Erat_2Erat_add\ V0a)\ V1b) = (ap\ (ap\ c.2Erat_2Erat_add\ V1b)\ V0a)))) \quad (42)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat.(\forall V1r2 \in ty_2Erat_2Erat.((ap\ (ap\ c.2Erat_2Erat_sub\ V0r1)\ V1r2) = (ap\ (ap\ c.2Erat_2Erat_add\ V0r1)\ (ap\ c.2Erat_2Erat_ainv\ V1r2)))))) \quad (43)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat.((ap\ c.2Erat_2Erat_ainv\ (ap\ c.2Erat_2Erat_ainv\ V0r1)) = V0r1)) \quad (44)$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& ((ap\ c_2Erat_2Erat_ainv\ (ap\ (ap\ c_2Erat_2Erat_add\ V0r1)\ V1r2)) = \\
& (ap\ (ap\ c_2Erat_2Erat_add\ (ap\ c_2Erat_2Erat_ainv\ V0r1))\ (ap \\
& \quad c_2Erat_2Erat_ainv\ V1r2))))))
\end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{46}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B)))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{49}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& (\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\
& (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\
& ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))))) \quad (54)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q)) \vee \neg(p \ V0p)))))) \quad (55)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& (\forall V2r3 \in ty_2Erat_2Erat. ((p \ (ap \ (ap \ c_2Erat_2Erat_les \\
& (ap \ (ap \ c_2Erat_2Erat_sub \ V0r1) \ V1r2)) \ V2r3)) \Leftrightarrow (p \ (ap \ (ap \ c_2Erat_2Erat_les \\
& \ V0r1) \ (ap \ (ap \ c_2Erat_2Erat_add \ V1r2) \ V2r3)))))))))
\end{aligned}$$