

thm_2Erat_2ERAT__MINV__EQ__0
(TMTBTi3N5Hr55TSFbWJ3xkNAHmnrwZ577aH)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p (ap P x))$) of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A P))))$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \tag{1}$$

Let $ty_2Erat_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \tag{2}$$

Let $c_2Erat_2Erep_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \tag{3}$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (ap V0P (ap (c_2Emin_2E_40 A P))))))$

Definition 6 We define $c_2Erat_2Erep_rat$ to be $\lambda V0a \in ty_2Erat_2Erat.(ap (c_2Emin_2E_40 ty_2Efrac_2Efrac (ap V0a (ap (c_2Emin_2E_3D (2^{2^a})) (ap V0a (ap (c_2Emin_2E_40 A P)))))))$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{4}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{5}$$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (6)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (7)$$

Definition 7 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2EFST\ ty_2Efrac_2Efrac)\ f)$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (8)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (9)$$

Definition 8 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E40\ ty_2Einteger_2Eint)\ a)$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (10)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (11)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (12)$$

Definition 9 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 10 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint_neg\ T1)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (13)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (14)$$

Definition 11 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (16)$$

Definition 12 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.$

Definition 13 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 14 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 15 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V3t3 \in 2.V3t3))))))$

Definition 17 We define $c_2Einteger_2EABS$ to be $\lambda V0n \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V1t1 \in 2.V1t1))))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (17)$$

Definition 18 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2ESND\ t))$

Definition 19 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (18)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (19)$$

Definition 20 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ t)$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Definition 21 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E2B\ t))$

Definition 22 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 23 We define $c_2EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V1t1 \in 2.V1t1))))$

Definition 24 We define $c_Efrac_Efrac_sgn$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap\ c_2EintExtension_2ES$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))^(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \quad (21)$$

Definition 25 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (22)$$

Definition 26 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)}) \quad (23)$$

Definition 27 We define $c_2Efrac_2Efrac_minv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap\ c_2Efrac_2Eabs_frac$

Definition 28 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat^{(2^{ty_2Efrac_2Efrac})}) \quad (24)$$

Definition 29 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap\ c_2Erat_2Eabs_rat_CLASS$

Definition 30 We define $c_2Erat_2Erat_minv$ to be $\lambda V0r1 \in ty_2Erat_2Erat.(ap\ c_2Erat_2Eabs_rat\ (ap\ c_2E$

Definition 31 We define $c_2Efrac_2Efrac_1$ to be $(ap\ c_2Efrac_2Eabs_frac\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2E$

Definition 32 We define $c_2Erat_2Erat_1$ to be $(ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_1)$.

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))^(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \quad (25)$$

Definition 33 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 34 We define $c_2Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 35 We define $c_2Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 36 We define $c_Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 37 We define $c_Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 38 We define $c_Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap (ap (c_Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Let $c_2Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Earithmetic_2Enum_CASE\ A_27a \in ((A_27a^{(A_27a^{ty_2Enum_2Enum})})^{A_27a})^{ty_2Enum_2Enum} \quad (26)$$

Definition 39 We define $c_2Efrac_2Efrac_0$ to be $(ap\ c_2Efrac_2Eabs_frac\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Enum_2Enum))))$

Definition 40 We define $c_2Erat_2Erat_0$ to be $(ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_0)$.

Definition 41 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21))$

Definition 42 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). (ap\ (c_2Ebool_2E_21))$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (27)$$

Definition 43 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1R \in (A_27a^{A_27b}).$

Definition 44 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1a \in A_27a. \lambda V2b \in A_27a.$

Definition 45 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in (A_27a^{A_27b}).$

Definition 46 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in (A_27a^{A_27b}).$

Definition 47 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in (A_27a^{A_27b}).$

Definition 48 We define $c_2Erat_2Erat_of_num$ to be $(ap\ (ap\ (c_2Erelation_2EWFREC\ ty_2Enum_2Enum)))$

Definition 49 We define $c_2Efrac_2Efrac_mul$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. \lambda V1f2 \in ty_2Efrac_2Efrac. \lambda V2r \in 2.$

Definition 50 We define $c_2Erat_2Erat_mul$ to be $\lambda V0r1 \in ty_2Erat_2Erat. \lambda V1r2 \in ty_2Erat_2Erat. (ap\ (ap\ c_2Efrac_2Efrac_mul\ V0r1\ V1r2))$

Definition 51 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (\lambda V3t \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V3t))))))$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (37)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. ((V0x = V0x) \Leftrightarrow True)) \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (43)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat.(\forall V1b \in ty_2Erat_2Erat.((ap (ap c_2Erat_2Erat_mul V0a) V1b) = (ap (ap c_2Erat_2Erat_mul V1b) V0a)))) \quad (44)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat.((\neg(V0a = (ap c_2Erat_2Erat_of_num c_2Enum_2E0))) \Rightarrow ((ap (ap c_2Erat_2Erat_mul V0a) (ap c_2Erat_2Erat_minv V0a)) = (ap c_2Erat_2Erat_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \quad (45)$$

Assume the following.

$$(\neg((ap c_2Erat_2Erat_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO))) = (ap c_2Erat_2Erat_of_num c_2Enum_2E0))) \quad (46)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat.((ap (ap c_2Erat_2Erat_mul (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) V0r1) = (ap c_2Erat_2Erat_of_num c_2Enum_2E0))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow False))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow False) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \Leftrightarrow (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \vee (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \Rightarrow (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\neg(p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \quad (56)$$

Theorem 1

$$(\forall V0r \in ty_2Erat_2Erat.((\neg(V0r = (ap \ c_2Erat_2Erat_of_num \ c_2Enum_2E0))) \Rightarrow (\neg((ap \ c_2Erat_2Erat_minv \ V0r) = (ap \ c_2Erat_2Erat_of_num \ c_2Enum_2E0))))))$$