

thm\_2Erat\_2ERAT\_\_MINV\_\_LES  
(TMNJ45mJfEURNSc9Lhyixinvyt5SCWtB5VW)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge P x)$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A P))))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 5** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 6** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 7** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (c\_2Ebool\_2E\_3F A P))))$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 9** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B$

**Definition 10** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \quad (7)$$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $ty\_2Efrac\_2Efrac : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \quad (9)$$

Let  $ty\_2Erat\_2Erat : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erat\_2Erat \quad (10)$$

Let  $c\_2Erat\_2Erep\_rat\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erat\_2Erep\_rat\_CLASS \in ((2^{ty\_2Efrac\_2Efrac})^{ty\_2Erat\_2Erat}) \quad (11)$$

**Definition 11** We define  $c\_2Erat\_2Erep\_rat$  to be  $\lambda V0a \in ty\_2Erat\_2Erat.(ap\ (c\_2Emin\_2E\_40\ ty\_2Efrac\_2Efrac$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (12)$$

Let  $c\_2Efrac\_2Erep\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Erep\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \quad (13)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (14)$$

**Definition 12** We define  $c\_2Efrac\_2Efrac\_dnm$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ t$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (15)$$

**Definition 13** We define  $c\_2Efrac\_2Efrac\_nmr$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty\_2Efrac\_2Efrac\_nmr\ f))$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint\_REP\_CLASS}) \quad (16)$$

**Definition 14** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ (c\_2Emin\_2E40\ ty\_2Einteger\_2Eint\_REP\ a))$

Let  $c\_2Einteger\_2Etint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{c\_2Einteger\_2Etint\_neg}) \quad (17)$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{c\_2Einteger\_2Etint\_eq}) \quad (18)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{c\_2Einteger\_2Eint\_ABS\_CLASS} \quad (19)$$

**Definition 15** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{c\_2Einteger\_2Eint\_ABS}$

**Definition 16** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap\ c\_2Einteger\_2Eint\_neg\ T1)$

**Definition 17** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 18** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ t1\ t2))\ (\lambda V2t \in 2.t))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (20)$$

**Definition 19** We define  $c\_2Epair\_2E2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ x\ y))$

Let  $c\_2Efrac\_2Eabs\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Eabs\_frac \in (ty\_2Efrac\_2Efrac)^{c\_2Efrac\_2Eabs\_frac} \quad (21)$$

**Definition 20** We define  $c\_2Efrac\_2Efrac\_ainv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_fn$   
Let  $c\_2Einteger\_2Etint\_mul : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum) \quad (22)$$

**Definition 21** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$   
Let  $c\_2Einteger\_2Etint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum) \quad (23)$$

**Definition 22** We define  $c\_2Einteger\_2Eint\_add$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

**Definition 23** We define  $c\_2Efrac\_2Efrac\_add$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

**Definition 24** We define  $c\_2Efrac\_2Efrac\_sub$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

**Definition 25** We define  $c\_2Erat\_2Erat\_equiv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

Let  $c\_2Erat\_2Eabs\_rat\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erat\_2Eabs\_rat\_CLASS \in (ty\_2Erat\_2Erat^{(2^{ty\_2Efrac\_2Efrac})}) \quad (24)$$

**Definition 26** We define  $c\_2Erat\_2Eabs\_rat$  to be  $\lambda V0r \in ty\_2Efrac\_2Efrac.(ap\ c\_2Erat\_2Eabs\_rat\_CLASS$

**Definition 27** We define  $c\_2Erat\_2Erat\_sub$  to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

**Definition 28** We define  $c\_2Erat\_2Erat\_ainv$  to be  $\lambda V0r1 \in ty\_2Erat\_2Erat.(ap\ c\_2Erat\_2Eabs\_rat\ (ap\ c\_2Erat$

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum) \quad (25)$$

**Definition 29** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

**Definition 30** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 31** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

**Definition 32** We define  $c\_2Einteger\_2EABS$  to be  $\lambda V0n \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND$

**Definition 33** We define  $c\_2EintExtension\_2ESGN$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap\ (c\_2Ebool$

**Definition 34** We define  $c\_2Efrac\_2Efrac\_sgn$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2EintExtension\_2ESGN$

**Definition 35** We define  $c\_2Efrac\_2Efrac\_minv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_fn$



Assume the following.

$$True \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee \neg(p V0t))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (36)$$

Assume the following.

$$((\forall V0t \in 2.(\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (37)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))) \quad (40)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). ((\neg(\forall V1x \in A\_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C))))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))) \quad (44)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. (((ap\ c\_2Einteger\_2Eint\_neg\ V0x) = (ap\ c\_2Einteger\_2Eint\_neg\ V1y)) \Leftrightarrow (V0x = V1y))) \quad (45)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat. ((ap\ (ap\ c\_2Erat\_2Erat\_sub\ (ap\ c\_2Erat\_2Erat\_of\_num\ c\_2Enum\_2E0))\ V0r1) = (ap\ c\_2Erat\_2Erat\_ainv\ V0r1))) \quad (46)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat. ((ap\ (ap\ c\_2Erat\_2Erat\_sub\ V0r1) (ap\ c\_2Erat\_2Erat\_of\_num\ c\_2Enum\_2E0)) = V0r1)) \quad (47)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat. ((ap\ c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Erat\_2Erat\_sgn\ (ap\ c\_2Erat\_2Erat\_ainv\ V0r1))) = (ap\ c\_2Erat\_2Erat\_sgn\ V0r1))) \quad (48)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.((\neg(V0r1 = (ap\ c\_2Erat\_2Erat\_of\_num\ c\_2Enum\_2E0))) \Rightarrow ((ap\ c\_2Erat\_2Erat\_sgn\ (ap\ c\_2Erat\_2Erat\_minv\ V0r1)) = (ap\ c\_2Erat\_2Erat\_sgn\ V0r1)))) \quad (49)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\neg(p\ (ap\ (ap\ c\_2Erat\_2Erat\_les\ V0r1)\ V0r1)))) \quad (50)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (p\ V1q) \Leftrightarrow (p\ V2r)) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (p\ V1q) \wedge (p\ V2r)) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (p\ V1q) \vee (p\ V2r)) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \quad (58)$$



Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q)) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q)) \vee \neg(p V0p)))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \quad (65)$$

**Theorem 1**

$$(\forall V0r1 \in ty\_2Erat\_2Erat. ((p (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) V0r1)) \Rightarrow (((p (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_minv V0r1)) (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0))) \Leftrightarrow (p (ap (ap c\_2Erat\_2Erat\_les V0r1) (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)))) \wedge ((p (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) (ap c\_2Erat\_2Erat\_minv V0r1))) \Leftrightarrow (p (ap (ap c\_2Erat\_2Erat\_les (ap c\_2Erat\_2Erat\_of\_num c\_2Enum\_2E0)) V0r1))))))$$