

thm_2Erat_2ERAT__MUL__CONG2 (TMW1hhmsefbhPuie51UQ6NU6BGwuAo9sKnd)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x)$

Let `ty_2Efrac_2Efrac` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Efrac_2Efrac} \tag{1}$$

Let `ty_2Erat_2Erat` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Erat_2Erat} \tag{2}$$

Let `c_2Erat_2Erep__rat__CLASS` : ι be given. Assume the following.

$$\text{c_2Erat_2Erep___rat___CLASS} \in ((2^{\text{ty_2Efrac_2Efrac}})^{\text{ty_2Erat_2Erat}}) \tag{3}$$

Definition 3 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 5 We define `c_2Erat_2Erep__rat` to be $\lambda V 0a \in \text{ty_2Erat_2Erat}. (\text{ap } (\text{c_2Emin_2E_40 } \text{ty_2Efrac_2Efrac}))$

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Einteger_2Eint} \tag{4}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A 0 A 1) \tag{5}$$

Let `c_2Efrac_2Erep__frac` : ι be given. Assume the following.

$$\text{c_2Efrac_2Erep___frac} \in ((\text{ty_2Epair_2Eprod } \text{ty_2Einteger_2Eint } \text{ty_2Einteger_2Eint})^{\text{ty_2Efrac_2Efrac}}) \tag{6}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c.2Epair_2ESND \\ A.27a\ A.27b \in (A.27b)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \end{aligned} \quad (7)$$

Definition 6 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2ESND\ ty$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (8)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (9)$$

Definition 7 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E.40\ (ty$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (10)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (11)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (12)$$

Definition 8 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 9 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c.2Epair_2EFST \\ A.27a\ A.27b \in (A.27a)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \end{aligned} \quad (13)$$

Definition 10 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2EFST\ ty$

Definition 11 We define $c_2Emin_2E.3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E.2F.5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E.21\ 2)\ (\lambda V2t \in$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (14)$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2E$

Let $c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)}) \quad (15)$$

Definition 14 We define $c_2Efrac_2Efrac_mul$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. \lambda V1f2 \in ty_2Efrac_2Efrac.$

Definition 15 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac. \lambda V1f2 \in ty_2Efrac_2Efrac.$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat^{(2^{ty_2Efrac_2Efrac})}) \quad (16)$$

Definition 16 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac. (ap\ c_2Erat_2Eabs_rat_CLASS$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\begin{aligned} (\forall V0a \in ty_2Efrac_2Efrac. (\forall V1b \in ty_2Efrac_2Efrac. \\ ((ap\ (ap\ c_2Efrac_2Efrac_mul\ V0a)\ V1b) = (ap\ (ap\ c_2Efrac_2Efrac_mul \\ V1b)\ V0a)))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty_2Efrac_2Efrac. (\forall V1y \in ty_2Efrac_2Efrac. \\ ((ap\ c_2Erat_2Eabs_rat\ (ap\ (ap\ c_2Efrac_2Efrac_mul\ (ap\ c_2Erat_2Erep_rat \\ (ap\ c_2Erat_2Eabs_rat\ V0x)))\ V1y)) = (ap\ c_2Erat_2Eabs_rat\ (\\ ap\ (ap\ c_2Efrac_2Efrac_mul\ V0x)\ V1y)))))) \end{aligned} \quad (20)$$

Theorem 1

$$\begin{aligned} (\forall V0x \in ty_2Efrac_2Efrac. (\forall V1y \in ty_2Efrac_2Efrac. \\ ((ap\ c_2Erat_2Eabs_rat\ (ap\ (ap\ c_2Efrac_2Efrac_mul\ V0x)\ (ap \\ c_2Erat_2Erep_rat\ (ap\ c_2Erat_2Eabs_rat\ V1y)))) = (ap\ c_2Erat_2Eabs_rat \\ (ap\ (ap\ c_2Efrac_2Efrac_mul\ V0x)\ V1y)))))) \end{aligned}$$