

thm_2Erat_2ERAT__RDIV__EQ
(TMSn2c2PzaVUqhfSKL1JGLw9AmRcUXACKBQ)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_21$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 10 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 11 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (7)$$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (8)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \quad (11)$$

Let $c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)}) \quad (12)$$

Definition 13 We define $c_2Efrac_2Efrac_1$ to be $(ap\ c_2Efrac_2Eabs_frac\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (13)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (14)$$

Definition 14 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND t$
Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2EFST \\ A.27a A.27b \in (A.27a^{(ty_2Epair_2Eprod A.27a A.27b)}) \end{aligned} \quad (15)$$

Definition 15 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2EFST ty$
Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (16)$$

Definition 16 We define $c_2Emin_2E.40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$
of type $\iota \Rightarrow \iota$.

Definition 17 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E.40 t$
Let $c_2Einteger_2Eint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (17)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (18)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})} \quad (19)$$

Definition 18 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)$

Definition 19 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 20 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Let $ty_2Erat_2Erat : \iota$ be given. Assume the following.

$$nonempty ty_2Erat_2Erat \quad (20)$$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat)^{(2^{ty_2Efrac_2Efrac})} \quad (21)$$

Definition 21 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap c_2Erat_2Eabs_rat_CLASS r)$

Definition 22 We define $c_2Erat_2Erat_1$ to be $(ap c_2Erat_2Eabs_rat c_2Efrac_2Efrac_1)$.

Let $c_2Erat_2Erep_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (22)$$

Definition 23 We define $c_2Erat_2Erep_rat$ to be $\lambda V0a \in ty_2Erat_2Erat.(ap (c_2Emin_2E40 ty_2Efrac$

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum))^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (23)$$

Definition 24 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteg$

Definition 25 We define $c_2Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 26 We define $c_2Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 27 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 28 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 29 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a}) A$

Let $c_2Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Earithmetic_2Enum_CASE A_27a \in (((A_27a^{(A_27a^{ty_2Enum_2Enum})})^{A_27a})^{ty_2Enum_2Enum}) \quad (24)$$

Definition 30 We define $c_2Efrac_2Efrac_0$ to be $(ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E2C ty_2$

Definition 31 We define $c_2Erat_2Erat_0$ to be $(ap c_2Erat_2Eabs_rat c_2Efrac_2Efrac_0)$.

Definition 32 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E$

Definition 33 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E40$

Definition 34 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E21$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2EARB A_27a \in A_27a \quad (25)$$

Definition 35 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 36 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1f$

Definition 37 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b$

Definition 38 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 39 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 40 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 41 We define $c_2Erat_2Erat_of_num$ to be $(ap (ap (c_2Erelation_2EWFREC ty_2Enum_2Enum$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod ty_2Enum_2Enum \\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \end{aligned} \quad (26)$$

Definition 42 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint$

Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (27)$$

Definition 43 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 44 We define $c_2Einteger_2EABS$ to be $\lambda V0n \in ty_2Einteger_2Eint.(ap (ap (ap (c_2Ebool_2E$

Definition 45 We define $c_2EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint.(ap (ap (ap (c_2Eboo$

Definition 46 We define $c_2Efrac_2Efrac_sgn$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_2EintExtension_2ES$

Definition 47 We define $c_2Efrac_2Efrac_minv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_2Efrac_2Eabs_f$

Definition 48 We define $c_2Erat_2Erat_minv$ to be $\lambda V0r1 \in ty_2Erat_2Erat.(ap c_2Erat_2Eabs_rat (ap c$

Definition 49 We define $c_2Efrac_2Efrac_mul$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2E$

Definition 50 We define $c_2Erat_2Erat_mul$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 51 We define $c_2Efrac_2Efrac_div$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2E$

Definition 52 We define $c_2Erat_2Erat_div$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap c$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (30)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat.(\forall V1b \in ty_2Erat_2Erat.(\forall V2c \in ty_2Erat_2Erat.((ap (ap c_2Erat_2Erat_mul V0a) (ap (ap c_2Erat_2Erat_mul V1b) V2c)) = (ap (ap c_2Erat_2Erat_mul (ap (ap c_2Erat_2Erat_mul V0a) V1b)) V2c)))))) \quad (34)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat.((ap (ap c_2Erat_2Erat_mul V0a) (ap c_2Erat_2Erat_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) = V0a)) \quad (35)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat.((\neg(V0a = (ap c_2Erat_2Erat_of_num c_2Enum_2E0)))) \Rightarrow ((ap (ap c_2Erat_2Erat_mul V0a) (ap c_2Erat_2Erat_minv V0a)) = (ap c_2Erat_2Erat_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \quad (36)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat.((\neg(V0a = (ap c_2Erat_2Erat_of_num c_2Enum_2E0)))) \Rightarrow ((ap (ap c_2Erat_2Erat_mul (ap c_2Erat_2Erat_minv V0a) V0a) = (ap c_2Erat_2Erat_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \quad (37)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat.(\forall V1r2 \in ty_2Erat_2Erat.((ap (ap c_2Erat_2Erat_div V0r1) V1r2) = (ap (ap c_2Erat_2Erat_mul V0r1) (ap c_2Erat_2Erat_minv V1r2)))))) \quad (38)$$

Theorem 1

$$(\forall V0r1 \in ty_2Erat_2Erat.(\forall V1r2 \in ty_2Erat_2Erat.(\forall V2r3 \in ty_2Erat_2Erat.((\neg(V2r3 = (ap c_2Erat_2Erat_of_num c_2Enum_2E0)))) \Rightarrow ((V0r1 = (ap (ap c_2Erat_2Erat_div V1r2) V2r3)) \Leftrightarrow ((ap (ap c_2Erat_2Erat_mul V0r1) V2r3) = V1r2))))))$$