thm_2Erat_2ERAT__SAVE__TO__CONS (TMLfKgnJvLncSfxWikVxFS725KnPfHgW3yN)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 4 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 5 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 6 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 7 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

Definition 9 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 10 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \quad (7)$$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (8)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (9)$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \quad (10)$$

Definition 11 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty_2Einteger_2Eint.(ap\ ($c_2Emin_2E_40 $(t$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \quad (11)$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \quad (12)$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \quad (13)$$

Definition 12 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 13 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Definition 14 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 15 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{14}$$

Definition 16 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $ty\_2Efrac\_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{15}$$

Let $c\_2Efrac\_2Eabs\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Eabs\_\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)}) \tag{16}$$

Definition 17 *We define* c_2Efrac_2Efrac__save *to be* $\lambda V0nmr \in ty\_2Einteger\_2Eint.\lambda V1dnm \in ty\_2Enur$

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{17}$$

Definition 18 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Definition 19 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 20 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 21 *We define* c_2Einteger_2Eint__le *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2E$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{18}$$

Definition 22 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 23 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 24 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 25 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{19}$$

Definition 26 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Definition 27 *We define* c_2Enumeral_2EiiSUC *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap$ c_2Enum_2ESUC $(ap$

Definition 28 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 29 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Efrac\_2Erep\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Erep\_\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{20}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{21}$$

Definition 30 *We define* c_2Efrac_2Efrac__dnm *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ t$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{22}$$

Definition 31 *We define* c_2Efrac_2Efrac__nmr *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty$

Definition 32 *We define* c_2Einteger_2EABS *to be* $\lambda V0n \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap$ (c_2Ebool_2EC

Definition 33 *We define* c_2EintExtension_2ESGN *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap$ (c_2Eboo

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{23}$$

Definition 34 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Eintege$

Definition 35 *We define* c_2Erat_2Erat__equiv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Let $ty\_2Erat\_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erat\_2Erat \tag{24}$$

Let $c\_2Erat\_2Eabs\_\_rat\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Eabs\_\_rat\_\_CLASS \in (ty\_2Erat\_2Erat^{(2^{ty\_2Efrac\_2Efrac})}) \tag{25}$$

Definition 36 *We define* c_2Erat_2Eabs__rat *to be* $\lambda V0r \in ty\_2Efrac\_2Efrac.(ap\ c\_2Erat\_2Eabs\_\_rat\_\_C1$

Definition 37 *We define* c_2Erat_2Erat__cons *to be* $\lambda V0nmr \in ty\_2Einteger\_2Eint.\lambda V1dnm \in ty\_2Einteg$

Assume the following.

$$True \tag{26}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{32}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))))) \tag{33}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{36}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg (p\ V0A)) \vee (p\ V1B))))) \tag{40}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg (p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2)))) \tag{43}$$

Assume the following.

$$(\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1b))\Rightarrow((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_nmr}\ (ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V0a)\ V1b))) = V0a)))) \tag{44}$$

Assume the following.

$$(\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1b))\Rightarrow((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_dnm}\ (ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V0a)\ V1b))) = V1b)))) \tag{45}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ V0x)\ V1y))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V1y))))) \tag{46}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ V0x)\ V1y))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1y)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x))))))) \tag{47}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Einteger\_2Eint}).((\forall V1n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))))\Leftrightarrow(\forall V2x \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V2x))\Rightarrow(p\ (ap\ V0P\ V2x)))))) \tag{48}$$

Assume the following.

$$(\forall V0c \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint.(\forall V2y \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0c)\ V1x)))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ V2y)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1x)))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0c))\ V2y)))\Leftrightarrow False)))))) \tag{49}$$

Assume the following.

$$(\forall V0y \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1x)\ V0y) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0y)\ V1x)))) \tag{50}$$

Assume the following.

$$(\forall V0z \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V2x)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1y)\ V0z)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V2x)\ V1y))\ V0z))))) \tag{51}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = V0x)) \tag{52}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V0x) = V0x)) \tag{53}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) = V0x)) \tag{54}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2z \in ty\_2Einteger\_2Eint.(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ V2z) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1y)\ V2z)) \Leftrightarrow (V0x = V1y))))) \tag{55}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1y))))) \tag{56}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{57}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{58}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x))\ V1y)))) \tag{59}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1y))))) \tag{60}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V1y)))) \tag{61}$$

Assume the following.

$$((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \tag{62}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2z \in ty\_2Einteger\_2Eint.(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V1y) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V2z)) \Leftrightarrow ((V0x = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \lor (V1y = V2z)))))) \tag{63}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)) \Leftrightarrow (V0m = V1n)))) \tag{64}$$

Assume the following.

$$
\begin{gathered}
(\forall V0p \in \mathit{ty\_2Einteger\_2Eint}.(\forall V1n \in \mathit{ty\_2Enum\_2Enum}. \\
(\forall V2m \in \mathit{ty\_2Enum\_2Enum}.(((\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_add} \\
(\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V0p) = V0p)\wedge(((\\
\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0p)\ (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num} \\
\mathsf{c\_2Enum\_2E0})) = V0p)\wedge(((\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num} \\
\mathsf{c\_2Enum\_2E0})) = (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\wedge \\
(((\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0p)) = \\
V0p)\wedge(((\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)))\ (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2m))) = (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
V1n)\ V2m)))))\wedge(((\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)))\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_neg} \\
(\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V2m)))) = (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ebool\_2ECOND}\ \mathit{ty\_2Einteger\_2Eint})\ (\mathit{ap} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V2m)\ V1n))\ (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}\ V1n) \\
V2m))))\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}\ V2m) \\
V1n))))))\wedge(((\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_neg} \\
(\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V1n))))\ (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V2m))) = (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ebool\_2ECOND}\ \mathit{ty\_2Einteger\_2Eint})\ (\mathit{ap}\ (\\
\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2m))\ (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}\ V2m) \\
V1n))))\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}\ V1n) \\
V2m))))))\wedge((\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_neg} \\
(\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V1n))))\ (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2m)))) = (\mathit{ap}\ \mathsf{c\_2Einteger\_2Eint\_\_neg} \\
(\mathit{ap}\ \mathit{c\_2Einteger\_2Eint\_\_of\_\_num}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V1n)\ V2m)))))))))))))))
\end{gathered}
\tag{65}
$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow False)\wedge (((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m)))))\Leftrightarrow False)\wedge((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m)))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n)))))))))))))) \tag{66}$$

Assume the following.

$$
\begin{gathered}
(\forall V0n \in \mathit{ty\_2Enum\_2Enum}.(\forall V1m \in \mathit{ty\_2Enum\_2Enum}.(\\
((\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V0n)) = V0n) \wedge (((\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = V0n) \wedge (((\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\\
\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (\\
\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (\\
\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V0n)\ V1m)))) \wedge \\
(((\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V0n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V0n)\ V1m)))) \wedge (\\
((\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V0n)\ V1m)))) \wedge (\\
((\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V0n)\ V1m)))) \wedge (\\
((\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V0n)) = (\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = (\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((\mathit{ap} \\
\mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V0n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V0n)\ V1m)))) \wedge (\\
((\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V0n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V0n)\ V1m)))) \wedge (\\
((\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V0n)\ V1m)))) \wedge (\\
((\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V0n)\ V1m)))) \wedge \\
(((\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V0n)) = (\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiiSUC} \\
(\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = (\\
\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (\mathit{ap}\ (\\
\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V1m))) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
V0n)\ V1m)))) \wedge (((\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
V0n)\ V1m)))) \wedge (((\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
V0n)\ V1m)))) \wedge ((\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
V0n)\ V1m)))))))))))))))))))))))))
\end{gathered}
\tag{67}
$$

Assume the following.

$$(\forall V0f1 \in ty\_2Efrac\_2Efrac.(\forall V1f2 \in ty\_2Efrac\_2Efrac.(((ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V0f1) = (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V1f2))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_equiv}\ V0f1)\ V1f2))))) \tag{68}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{69}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{70}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{71}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{72}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{73}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{74}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{75}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{77}$$

Theorem 1

$$(\forall V0a1 \in ty\_2Einteger\_2Eint.(\forall V1b1 \in ty\_2Enum\_2Enum.$$
$$((ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_save}\ V0a1)\ V1b1)) =$$
$$(ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_cons}\ V0a1)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1b1))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))))$$