

thm_2Erat_2ERAT__SGN__AINV__RWT (TMH- pxCapMTTVhTmUWbKaAXbxek6CL7449tb)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2EBOUNDED$ to be $(\lambda V0v \in 2.c_2Ebool_2ET)$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 4 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{4}$$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \tag{5}$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 6 We define $c_2Emarker_2ECong$ to be $\lambda V0x \in 2.V0x$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_Ebool_E_F5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21 2) (\lambda V2t \in 2.$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty ty_2Efrac_2Efrac \quad (7)$$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod ty_2Einteger_2Eint ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (8)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2ESND A.27a A.27b \in (A.27b^{(ty_2Epair_2Eprod A.27a A.27b)}) \quad (9)$$

Definition 9 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND ty$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2EFST A.27a A.27b \in (A.27a^{(ty_2Epair_2Eprod A.27a A.27b)}) \quad (10)$$

Definition 10 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2EFST ty$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (11)$$

Definition 11 We define $c_2Emin_E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_E_40 (t$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{ty_2Einteger_2Eint})^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})}) \quad (12)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (13)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})} \quad (14)$$

Definition 13 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 14 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 15 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 16 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a})$

Let $ty_2Erat_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \quad (15)$$

Let $c_2Erat_2Erep_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (16)$$

Definition 17 We define $c_2Erat_2Erep_rat$ to be $\lambda V0a \in ty_2Erat_2Erat.(ap\ (c_2Emin_2E40\ ty_2Efrac_2Efrac))$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (17)$$

Definition 18 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint)$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (18)$$

Definition 19 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod))$

Let $c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac)^{(ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)} \quad (19)$$

Definition 20 We define $c_2Efrac_2Efrac_ainv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap\ c_2Efrac_2Eabs_frac)$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat)^{(2^{ty_2Efrac_2Efrac})} \quad (20)$$

Definition 21 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap\ c_2Erat_2Eabs_rat_CLASS)$

Definition 22 We define $c_2Erat_2Erat_ainv$ to be $\lambda V0r1 \in ty_2Erat_2Erat.(ap\ c_2Erat_2Eabs_rat\ (ap\ c_2Efrac_2Efrac_ainv))$

Definition 23 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (21)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (22)$$

Definition 24 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (23)$$

Definition 25 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 26 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (24)$$

Definition 27 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 28 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 29 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 30 We define $c_2EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ (c_2Ebool$

Definition 31 We define $c_2Efrac_2Efrac_sgn$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap\ c_2EintExtension_2ES$

Definition 32 We define $c_2Erat_2Erat_sgn$ to be $\lambda V0r \in ty_2Erat_2Erat.(ap\ c_2Efrac_2Efrac_sgn\ (ap\ c_2$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$(\forall V0v \in 2.((p\ (ap\ c_2Ebool_2EBOUNDED\ V0v)) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$(\forall V0f1 \in ty_2Efrac_2Efrac.((ap\ c_2Efrac_2Efrac_ainv\ (ap\ c_2Efrac_2Efrac_ainv\ V0f1)) = V0f1)) \quad (29)$$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\ & (((ap\ c_2Einteger_2Eint_of_num\ V0m) = (ap\ c_2Einteger_2Eint_of_num \\ & \quad V1n)) \Leftrightarrow (V0m = V1n)))) \wedge ((\forall V2x \in ty_2Einteger_2Eint.(\forall V3y \in \\ & ty_2Einteger_2Eint.(((ap\ c_2Einteger_2Eint_neg\ V2x) = (ap\ c_2Einteger_2Eint_neg \\ & \quad V3y)) \Leftrightarrow (V2x = V3y)))) \wedge (\forall V4n \in ty_2Enum_2Enum.(\forall V5m \in \\ & ty_2Enum_2Enum.(((ap\ c_2Einteger_2Eint_of_num\ V4n) = (ap \\ & \quad c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num\ V5m))) \Leftrightarrow \\ & ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))) \wedge (((ap\ c_2Einteger_2Eint_neg \\ & \quad (ap\ c_2Einteger_2Eint_of_num\ V4n)) = (ap\ c_2Einteger_2Eint_of_num \\ & \quad V5m)) \Leftrightarrow ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))))))))) \quad (30) \end{aligned}$$

Assume the following.

$$(p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ ty_2Efrac_2Efrac\ ty_2Erat_2Erat) \\ \quad c_2Erat_2Erat_equiv)\ c_2Erat_2Eabs_rat)\ c_2Erat_2Erep_rat))) \quad (31)$$

Assume the following.

$$(\forall V0x \in ty_2Efrac_2Efrac.((ap\ c_2Erat_2Eabs_rat\ (ap\ c_2Efrac_2Efrac_ainv\ (ap\ c_2Erat_2Erep_rat\ (ap\ c_2Erat_2Eabs_rat\ V0x)))) = (ap\ c_2Erat_2Eabs_rat\ (ap\ c_2Efrac_2Efrac_ainv\ V0x)))) \quad (32)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat.((ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Erat_2Erat_sgn\ (ap\ c_2Erat_2Erat_ainv\ V0r1))) = (ap\ c_2Erat_2Erat_sgn\ V0r1))) \quad (33)$$

Theorem 1

$$(\forall V0r \in ty_2Erat_2Erat.((ap\ c_2Erat_2Erat_sgn\ (ap\ c_2Erat_2Erat_ainv\ V0r)) = (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Erat_2Erat_sgn\ V0r))))$$